Who Pays? Inefficiencies Arising from Pressure in Joint Liability Lending Microfinance Programs

Abhishek Ananth, Department of Economics, Cornell University

December 3, 2020

Please find latest version here.

Abstract

I present a game theoretic model of Joint Liability Lending (JLL) microfinance programs with endogenous peer pressure to repay. In addition, I describe a role for institutional pressure applied by microfinance institutions (MFIs). This model helps better explain two important empirical findings in the literature. Firstly, observed repayment rates in not for profit microfinance programs are very high. Secondly, the welfare implications of these programs (as evidenced by RCTs) are small. I analyze a sequential game where the MFI's interest rate, the projects selected by the group members and the subsequent peer pressure and repayment decision are endogenized. I characterize the solutions and analyze the game in numerical examples.

The most striking intuition generated by the model is that when (risk-averse) households can choose between low risk-low reward and high risk-high reward investments, and the MFI prefers to set low interest rates, the resulting equilibrium boasts inefficiently high repayment rates. This leads to an inefficient transfer of the burden of risk bearing onto the households who respond by inefficiently choosing low risk-low reward investments. Thus, counter to the main purpose of these programs of poverty alleviation, this implies that growth generating investments (high risk-high reward) are left under funded in equilibrium. Thus, the model provides a more satisfactory explanation of some of the empirical findings in this literature.

Acknowledgements: The author is grateful for all the helpful comments and suggestions from Francesca Molinari, Larry Blume, David Easley, Kaushik Basu, Johnathan Murdoch, Jeanne Lafortune as well as participants at WEAI 2019 and NEUDC 2019. All errors that remain are author's own, likely despite warnings by the aforementioned.

"... top-down repayment pressure can lead to forms of borrower discipline which are unnecessarily exclusionary, and which can contradict the broader (social) aims of solidarity group lending"

— Richard Montgomery

"... despite the promise of microcredit as a development tool aimed at helping borrowers escape poverty, there was a general consensus among high-level staff that credit in and of itself was not sufficient to achieve Fundacion Paraguaya's mission to alleviate poverty."

— Caroline E. Schuster

1 Introduction

In this paper, I consider the role of top down pressure in encouraging repayment and project (investment) selection in Joint Liability Lending microfinance programs (*group loan*). By top down pressure, I denote the two types of pressure to repay observed to be in action in these programs: pressure to repay from the Microfinance Institution (MFI) *institutional pressure*, and endogenous pressure by group members *peer pressure*. In doing so, I discuss its implications on how the risk is distributed between the group members and the MFI. Finally, I demonstrate the effect of excessive risk on group members on their project selection.

Joint Liability Lending (JLL) has been an exceedingly common form of loan contract offered in the early days of microfinance programs world over. Even today, while many lenders offer a large menu of contracts, JLL contracts remain a popular choice in lending to the poorest among the MFI's clientele (those who do not own land, for example) or to newer clients. The basic premise is that participating households organize into groups to receive a joint liability loan. In such a program, the obligation of repaying the loan falls collectively on the group and not on individual members. If the group fails to repay, the microfinance institution refuses to offer all members of the group a loan thereafter. Thus, side-payments among group members are incentivized in the event of bankruptcy of few members of the group. While microfinance has undergone rapid commercialization in the recent years, at its inceptions, popular stalwarts operated as non-profits (sometimes even under heavy subsidization from the government). This paper focuses on microfinance institutions (MFIs) that operate as non-profits (or NGOs) and not as profit seeking enterprises. Grameen bank, BRAC and the early days of SKS are all examples of microfinance programs that operated as non-profits.

This paper explores the role of top-down pressure in rationalizing an empirical paradox observed among microfinance programs. Firstly, repayment rates are predominantly high with most instances of microfinance resulting in very high repayment rates; between 89% and 98% [1]. Secondly, while some of the empirical evidence on the effectiveness of microfinance in alleviating poverty is controversial at best, evidence from randomized control trials suggests a small, if not insignificant, average effect of these programs on a variety of welfare indicators such as profitability of business, health and women empowerment.

[1]A notable counter-example is Andhra Pradesh where repayment rates plummeted to around 10 - 15% when the program was commercialized (operated *for profit*).

Within the purview of the existing models, these empirical findings suggest that relaxing credit constraints might not be of paramount concern for policy makers interested in alleviating poverty. This is because the existing literature rationalizes the high repayment rates through an efficient allocation of resources, that is, JLL addresses problems of adverse selection and moral hazard it was designed to overcome. The lack of evidence on poverty alleviation seems to suggest that even an efficient alleviation of credit constraints does not help these local economies grow. This offers a rather grim outlook for prospects of poverty alleviation by means of relaxing the credit constraints. It would suggest that relaxing the credit constraint in these markets is inconsequential if it is not foreshadowed by investing in building human capital and/or other forms of investment necessary for incubating and developing profitable ventures. As one suspects, such multi-pronged interventions are not only resource intensive but would also need to be carefully curated with the local context in mind, rather than allowing the market to leverage its local information to do so organically.

In this paper, I present a canonical model of group lending with two group members and a *not for profit* MFI. The MFI decides whether to operate or not and the interest rate it would offer the group. Group members simultaneously choose their projects (investments) and subsequently play the group repayment group. In this group repayment game, for any realization of project outcomes, members simultaneously decide whether or not to pressure to each and whether or not to repay thereafter. Thus, the model allows group members to apply (costly) *peer pressure* to facilitate *good repayment behavior*. Here, good repayment is simply a *social norm* that specifies how much each member of a group repays in different states of the world while peer pressure denotes the threat of severing social ties across which households share resources in a mutually beneficial way. The model also details the strategic choices of the MFI. I argue that MFIs invest in building relationships with their clients. This relationship allows them some control over the repayment decisions of the members. In the model, I suggest that this *institutional pressure* allows the market to *select* the equilibrium with the highest repayment in the event of multiple equilibria. Finally, I describe and solve the social planner's problem in this environment, allowing welfare analysis.

A striking result generated by the model is that the group repayment game contains equilibrium with inefficiently high repayment rates supported by very high pressure. Consequently, well-meaning MFI that steer the market towards these equilibria in an attempt to keep interest rates low (and hence loans accessible), end up inefficiently transferring the burden of risk bearing onto group members. Risk averse members now respond by inefficiently choosing low risk-low reward investments. This is detrimental to the economy since it implies that growth generating investments (high risk-high reward) are left under funded in equilibrium. Thus, this model provides an alternate explanation of the empirical findings that suggests that the very feature of group lending programs (high repayment) often heralded as a celebration of the success of these programs might also be deterring its participants from achieving the desired outcome.

2 Literature review

The literature on the theory of group lending in microfinance programs has established how social collateral might replace financial capital. Stiglitz (1990) suggests a mechanism through which peer monitoring could alleviate problems of moral hazard. In the setup therein, participants are allowed to choose between a safe and a risky project (investment). The paper suggests that while agents might prefer the risky project, the peer monitoring inherent in group lending allows banks to ensure that agents avoid the risky project. While this result seems qualitatively very similar to the result in this paper, it is worth noting that the risky project in Stiglitz (1990) has lower expected returns than the safe project. To the contrary, in this paper, the risky project will have a higher mean and variance compared to the safe project. Varian (1990) explores various aspects of popular microfinance programs and their impacts on alleviating moral hazard using the theory of mechanism design. Arnott, Stiglitz (1991) demonstrates how non-market insurance is beneficial when insurers can perfectly observe each other's effort. Thus, it suggests how peer monitoring, where peers are well informed about each other's project outcomes might help mitigate moral hazard. Ghatak (1999) demonstrates how group lending may leverage information on peers among participants through self-selection of groups to overcome problems of moral hazard. van Tassel (1999), Armendariz de Aghion, Gollier (1998), and Laffont, N'Guessan (1999) show qualitatively similar results in more more varied contexts. Ghatak, Guinnane (1999) explore the impact of screening, monitoring, state verification as well as preventing strategic defaults. Madajewicz (2011) talks about the coexistence of joint as well as individual liability lending contracts as a welfare-maximizing solution to the lending problem. Ahlin (2015) explores the role of group size in mitigating adverse selection. He finds that a larger group can be beneficial so long as the households have some information about their peers. More recently, the literature has turned its attention to the consequences of commercialization in this environment (see De Quidt, Ghatak (2018) and De Quidt, Fetzer, Ghatak (2018) for examples).

Much of the literature models group lending programs as contracts, much like traditional individual loans. The key distinction between the two contacts is that the group lending contract includes an additional joint liability payment for every peer whose investment has failed. Thus, the amount to be repaid in the event of non-bankruptcy is not fixed, and depends on the investment outcomes of peers in the group. However, any paper that seeks to understand the effect of pressure on said strategic repayment decisions must allow repayment decisions of peers to impact the payoff to a given household. Consequently, in this model, I formulate a repayment game where households, having observed outcomes of each other's projects simultaneously decide whether or not to apply pressure. Subsequently, they chose whether or not to repay. Besley, Coate (1995) is the closest in this regard. They set up a repayment game to endogenize strategic repayment decisions. However, while Besley, Coate (1995) consider the impact of exogenous peer punishment (termed "social collateral") on repayment decisions, this paper is interested in endogenizing this informal mechanism. Further, this paper also explores the effect of this pressure on choice of projects that households invest in.

Over the years, a large empirical literature has emerged exploring different facets of microfinance. Ahlin, Townsend (2007) conduct experiments on credit contracts offered in Thailand. They find no evidence to suggest group lending is more prevalent as correlation between projects increases. They do find evidence suggesting the prevalence of group lending is U-shaped in wealth level and increasing in dispersion of wealth as well as the assortative matching described by Ghatak (1999). Karlan (2007) finds evidence for the role of social ties among group members in reducing delinquencies using an RCT in Peru. Al-Azzam, Hill, Sarangi (2011) establish a similar finding in Jordan, they find that relegious ties are particularly relevant. Ahlin, Suandi (2019) show that while the overall share of group lending among microfinance programs has been declining, they are used more extensively in poorer communities. Banejree, Duflo, Glennerster, Kinnan (2015) estimate the intent-to-treat effect of JLL microfinance in India using an RCT. They find that while business profitability increased, there was no impact of consumptions, health, education and women's empowerment (all though to be good indicators of poverty alleviation). Creṕon, Devoto, Duflo, Parienté (2015), Angelucci, Karlan, Zinman (2015), Attanasio, Augsburg, De Haas, Fitzsimons, Harmgart (2015), Augsburg, De Haas, Harmgart, Meghir (2015) and Tarozzi, Desai, Johnson (2015) all suggest an small (if not insignificant) impact of microfinance programs on its principal goal of poverty alleviation. There is also a large and highly contentious literature which evaluates the effectiveness of microfinance in non-RCT settings. Pitt, Khandker (1998) examine data from 1800 households in over 80 villages in Bangladesh. Using the eligibility criterion of the Grameen Bank, they estimate statistically and economically significant effects of access to group loans on consumptions as well as poverty alleviation. While very influential, this paper was met with some heavy criticism in Roodman, Morduch (2013), which failed to replicate the findings in Pitt, Khandker (1998) is a similar model on the same data. This then led to an exchange of ideas between the authors in a series of papers.

3 Repayment game

3.1 Setup

In this section, I lay out our model for repayment decisions within the group. I consider a canonical group with two homogenous households A and B. These households are endowed with some fixed income of w per period. Households within the group are offered a loan of \$1 each at interest rate r (implying the group as a whole must repay $2r$). Each households invests its loan in a project which is modeled as a lottery. Specifically, the returns to the project is assumed to have the following probability law:

$$\text{return} = \begin{cases} R, & \text{with probability } \mu \\ 0, & \text{with probability } 1-\mu \end{cases}$$

where $\mu \in (0.5, 1)$ and $R > 1$. I make two assumptions here for tractability. Firstly, the project available to households is common knowledge. Secondly, the project outcomes are independent across households. Later, this assumption is relaxed. In this paper, I denote by $\lambda = (\lambda_A, \lambda_B) \in \{0,1\}^2$ the project outcome, where $\lambda_i = 1$ if the outcome is R (project has succeeded) and $\lambda_i = 0$ if the outcome is 0 (project has failed).

Upon realizing project outcomes, households decide whether or not the pressure each other into following good repayment behavior. Subsequently, households choose whether or not to follow good repayment behavior having observed whether or not their peer applies pressure. Here, good repayment behavior is simply a norm $\tilde{x}(.) : \{0,1\}^2 \to [0,1]^2$ that prescribes 'ideal' payment by each household in every state of the world (project outcome). I assume the following specification:

$$\tilde{x}_i(\lambda) = \begin{cases} \theta, \text{ if } \lambda_i = 0 \\ 1 + (1 - \lambda_j) \cdot (1 - \theta), \text{ if } \lambda_i = 1 \end{cases}$$

where $\theta \in (0, 0.5]$ is the share of loan a household whose investment has failed is expected to repay and $j = \{A, B\} \setminus \{i\}$. The above norm suggests that in the event that the projects of both households succeed, both households simply pay their dues (r each). In the event that one household succeeds while the other fails, the norm suggests that the failed household pays only a fraction θ of its dues. The remainder $1 - \theta$ fraction is made up from contributions of the successful peer in addition to the r it owes Finally, when both fail, even when all participants follow the norm, full repayment does not result. I will demonstrate that in such circumstances it is optimal for participants not to pressure each other and hence not make the repayment. The key advantage of using a norm to specify good repayment behavior is that repayment decisions are now binary: either pay the norm suggested amount or deviate and pay nothing. While this assumption does not simplify the game enough to generate uniqueness of equilibria, it makes the set of equilibria more tractable. Denote these binary repayment decisions by $d_j \in \{0,1\}$ where $j \in \{A, B\}$. In keeping with the on-field practice among microfinance programs, the successful repayment of the group loan ensures that the participants have continued access to this particular source of credit. In my model, $\phi(r)$ denotes a net-present value of having continued access to this source of credit (*continuation value*). I will elaborate on this once the full model has been established. Given that each household's repayment decision has a direct impact on the other's payoff, an innovation in this paper allows households to endogenously engage in peer pressure and monitoring. Here, peer pressure and monitoring are though to capture various social penalties that maybe levied by households onto their peers. One commonly documented example of this is making threats of punishment to discourage peers from reneging on repayments. In communities where such programs are often set-up, there exists a vibrant social culture of exchanging goods such as grains and commodities, or even farm equipment such as ploughs and shovels. Since groups in joint liability lending are usually self-selected, it is likely that the members of the group also engage in such mutually beneficial exchanges. In such an environment, the application of pressure corresponds to a household threatening its peer (with commitment) to stop sharing resources in the event that the peer does not repay its due (as suggested by the norm). This pressure decision is assumed to be binary and is denoted by $\delta_j \in \{0,1\}$ for all $j \in \{A, B\}$, where $\delta_j = 1$ corresponds to j applying pressure on $\{A, B\} \setminus \{j\}$ and $\delta_j = 0$ otherwise.

In summary, the model presented here begins with a group containing two households $\{A, B\}$. Upon receiving a loan of \$1 each, at gross interest rate r, both households invest in their projects. Having observed the outcome (λ) of the investment, both households simultaneously decide whether or not to apply pressure on each other (δ). I refer to this as *stage 1* of the repayment game. Finally, upon observing each other's pressure decisions, households simultaneously make repayment decisions (d). I refer to this as *stage 2* of the repayment game. With the notation established, I now discuss strategies and payoffs associated with the repayment game.

Definition 1 A *repayment play* for a given realization of investment outcomes, $\lambda \in \{0,1\}^2$, is a (pure) strategy profile listing the repayment decision of each household in each sub-game that may result after stage 1 (pressure). This will be denoted by $d^\lambda(.) : \{0,1\}^2 \to \{0,1\}^2$, the superscript is ignored when convenient to enhance readability.

Definition 2 A (pure) strategy profile of the *repayment game* lists for each realization of investment outcomes, $\lambda \in \{0,1\}^2$, a profile of pressure decisions, $\delta = (\delta_A^\lambda, \delta_B^\lambda) \in \{0,1\}^2$, and a consequent *repayment play* that determines repayment decisions in every sub-game that may result after the pressure stage, $d^\lambda(.) : \{0,1\}^2 \to \{0,1\}^2$. σ denotes a strategy profile $\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda\big)$.

The payoff to household i associated with strategy profile $\sigma = \big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda\big)$ is:

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda\big) = \sum_\lambda \mathbb{P}(\lambda) \times U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big)$$

where the state relevant payoff function is:

$$\begin{aligned} U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) = {} & u\big(w + R\lambda_i - r d_i^\lambda(\delta_A^\lambda, \delta_B^\lambda)\tilde{x}_i(\lambda)\big) \\ & + \phi(r) 1\Big\{ \sum_{j \in \{A,B\}} d_j^\lambda(\delta_A^\lambda, \delta_B^\lambda)\tilde{x}_j(\lambda) = M \Big\} \\ & - \gamma \max\Big\{ \delta_A^\lambda\big(1 - d_B^\lambda(\delta_A^\lambda, \delta_B^\lambda)\big), \delta_B^\lambda\big(1 - d_A^\lambda(\delta_A^\lambda, \delta_B^\lambda)\big) \Big\} \\ & - \eta\delta_i^\lambda \end{aligned}$$

The first term is simply the Bernoulli felicity function which depends on the initial wealth, investment outcome and repayment amount for the state. In this paper, I assume households are risk averse. Consequently, this felicity function is assumed to belong to the CARA family with risk aversion parameter α. As suggested earlier, the second term, $\phi(r) > 0$, corresponds to the net present value of having continued access to this source of credit in the event of full repayment of the group loan. The third term is the contingent cost of engaging in peer pressure, γ is the disutility associated with the severing of the relationship involving mutually beneficial exchanges between the households. This cost is only inflicted in the event that one households applies pressure while the other does not respond to this pressure by repaying its share as suggested by the norm. Finally, the last term η captures the cost of monitoring peers or the padlock cost of applying pressure (eg. cost of buying a padlock or the emotional cost of issuing a stern warning to a friend). It is assumed that $\gamma > \eta > 0$.

3.2 Equilibrium

Definition 3 *Sub-game perfect equilibrium in the repayment game.* A strategy profile, $\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda \in \{0,1\}^2\}$ is a sub-game perfect equilibrium if the following conditions hold:

1. $\forall\lambda,\ i \in \{A, B\}$, given $\delta \in \{0,1\}^2$:

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) \geq U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^{\lambda\prime}(\delta_A^\lambda, \delta_B^\lambda); \lambda\big)$$

   for any alternate repayment play: $d^{\lambda\prime} : \{0,1\}^2 \to \{0,1\}^2$

2. $\forall\lambda,\ i \in \{A, B\}$, given $d^\lambda(.)$:

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) \geq U_i\big(\delta_A^{\lambda\prime}, \delta_B^{\lambda\prime}, d^\lambda(\delta_A^{\lambda\prime}, \delta_B^{\lambda\prime}); \lambda\big)$$

Notice that the above restrictions need to hold for each realization of λ. In fact, owing to the simple and symmetric nature of the uncertainty in this context, a strategy $\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda \in \{0,1\}^2\}$ is a sub-game perfect equilibrium if and only if, for every realization of λ, the strategy $\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \}$ is sub-game perfect in the sub-game beginning at realization of investment outcomes λ. This simplifies the analysis of equilibria in this game. Based on the structure set up, I now discuss an intuitive result that arises in any sub-game perfect equilibrium.

Proposition 1 *No pressure wasted.* Suppose there is a sub-game perfect equilibrium $\sigma = \{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda \in \{0,1\}^2\}$ where for some λ, $\delta^\lambda \neq (0,0)$, then it must be that $d_A^\lambda(\delta^\lambda)\tilde{x}_A(\lambda) + d_B^\lambda(\delta^\lambda)\tilde{x}_B(\lambda) = 2$ i.e. full repayment occurs.

The above proposition basically states that if any household applies pressure in an equilibrium, then full repayment results in than equilibrium. This arises from the strictly positive cost of applying pressure, $\eta > 0$.

Remark: This proof employs the *one shot deviation principle*: a popular result that states that a strategy profile for finite extensive form game is a sub-game perfect equilibrium iff there exists no profitable deviation for every sub-game and for every player.

As is characteristic in extensive form games with strategic complementarity, there are a large number of sub-game perfect equilibria. In an attempt to make the analysis more tractable, I now make reasonable restrictions on the strategy space.

Assumption 1 *Monotone repayment response to peer pressure.* In any sub-game perfect equilibrium $\sigma = \{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda \in \{0,1\}^2\}$, $\forall\lambda \neq (0,0)$, if $\delta^\lambda > \delta^{\lambda\prime}$ (greater in the vector sense) in any sub-game perfect equilibrium, then $d^\lambda(\delta) \geq d^\lambda(\delta^{\lambda\prime})$ in that equilibrium.
Additionally, when $\lambda \neq (0,0)$, the only equilibrium play is $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$.

Essentially, the first part rules out repayment play where when additional household applies pressure, at least one household that would have otherwise repaid doesn't. The appendix contains an illustration of the type of behavior this assumption rules out. The second part of the assumption basically states that in the event that the investments of both participants fail, the *unique* equilibrium play that would result is that neither households applies any pressure and neither pays its share of the norm. This assumption is essentially without loss of generality. First of all, note that when $\lambda = (0,0)$, $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$ is trivially a best response since even full compliance with the norm doesn't lead to full repayment, thus from the contrapositive of the above lemma, neither participant has any incentive to apply pressure. Further note that in every equilibrium in the sub-game starting at this state, there will be no repayment and no pressure on the equilibrium path. This is formalized in the following proposition.

Proposition 2 When $\lambda = (0,0)$ one can assume $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$ is the only sub-game perfect equilibrium *without any loss of generality.*

Definition 4 *Admissible Equilibrium.* I denote any sub-game perfect equilibrium that satisfy the above assumption as an admissible equilibrium.

3.3 Characterizing the set of Admissible Equilibria

There are 4 states (investment outcome realizations) in this model: both households succeed, both fail, A succeeds while B fails and vice versa. Further, in each state that there are 256 distinct *repayment plays* that agents may choose. The following proposition recognizes that only a small subset of *repayment plays* are in fact sub-game perfect. In the two person context, out of the 256 candidate *repayment plays*, only 36 can be sustained.

Proposition 3 *Game implied restrictions.* Given the game described above, for all *continuation profiles* supported in any sub-game perfect equilibrium in sub-games with $\lambda \neq (0,0)$:

1. $d^\lambda(0,0) \in \{(0,0),\ (1,1)\}$
2. $d^\lambda(1,0) \neq (1,0)$ likewise $d^\lambda(0,1) \neq (0,1)$
3. $d^\lambda(1,1) \in \{(0,0),\ (1,1)\}$

Intuition The first assertion in this proposition states that in any optimal repayment play, partial repayment cannot occur when neither household applies any pressure, i.e. either both households or neither repay. This is a direct consequence of the strategic complementarity in repayments. Notice that in the event of partial repayment, neither household will be allowed to borrow from the microfinance institution thereafter. Thus, the household that is paying is only loosing money (be repaying now) and now getting any additional utility from continued access to this source of credit. So, in the event that this household is not under pressure to comply with the repayment norm, it must simply choose not to repay. The remaining assertions are more subtle. Collectively, they suggests that the act of applying pressure will not induce the household applying pressure to repay without also inducing the other household to repay. This is because the household applying pressure faces the same (if not lower) incentives to repay its share when it chooses to apply pressure. As a consequence of the non-repayment of the other household, the social penalty as well as the MFIs punishment (no access to future loans) are levied on the group regardless.

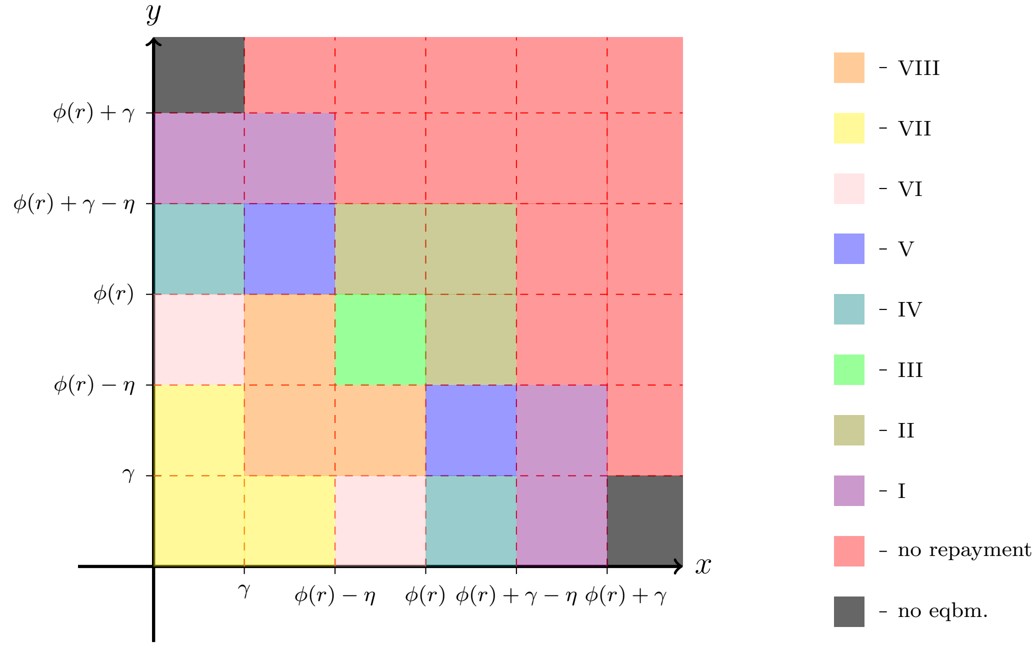
The following proposition discusses a similar result to the one above. It considers *repayment plays* that may be supported in an *admissible equilibrium*, i.e. satisfy *assumption 1* in addition to conditions required for sub-game perfection.

Proposition 4 Under *assumption 1*, for all $\lambda \neq (0,0)$, the number of optimal repayment plays reduces from 36 to 11.

This can be verified by running through all the 36 options and checking if they satisfy assumption 1. In the appendix, I list the 11 consistent *repayment plays* supported in an *admissible equilibrium*. For sub-games with $\lambda \neq (0,0)$, any of the 11 possible *repayment plays* listed above could be chosen. In the sub-game with $\lambda = (0,0)$, *assumption 1* provides the only *admissible strategy*. Consequently, there are a large number of possible *admissible equilibria*.

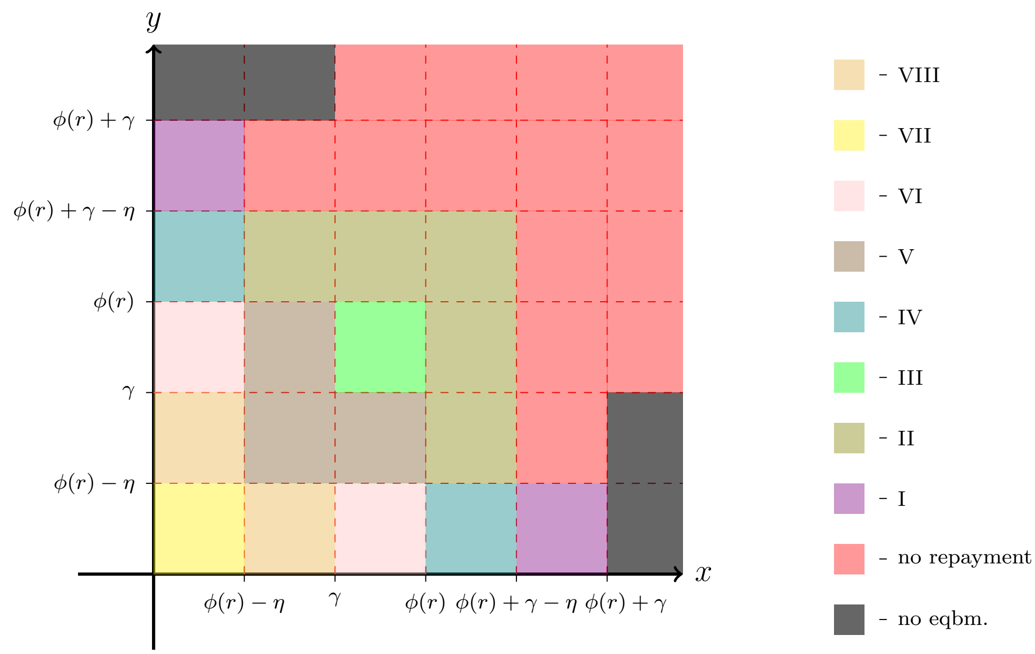
In what follows, I illustrate the actions observed on equilibrium path (for a given state, λ) based on parameter values. In the graphs that follow, the x-axis represents difference in the Bernoulli felicity from household A repaying the norm suggested amount, $X_A = u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) - u\big(w + R\lambda_A\big)$. Likewise, the y-axis represents the same for household B, $X_B = u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big)$.

Figure 1: $\phi(r)+\gamma > \phi(r)+\gamma-\eta > \phi(r) > \phi(r)-\eta > \gamma$



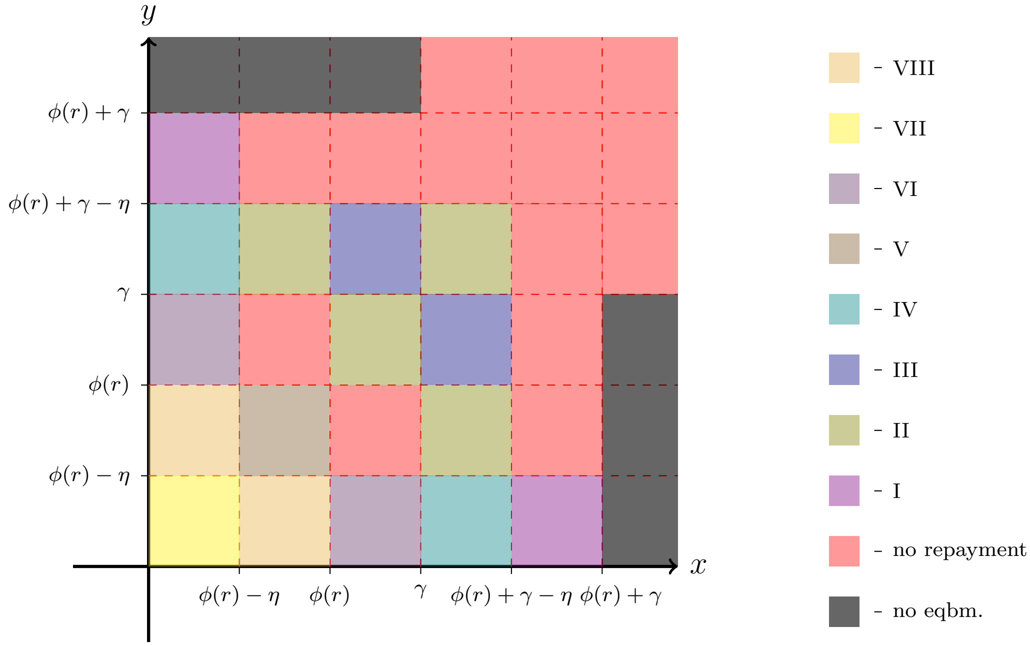
I = no repayment or repayment with one hh applying pressure; II = no repayment or repayment with both hh applying pressure; III = no repayment or repayment with no pressure or both hh applying pressure; IV = repayment with one or both hh applying pressure; V = no repayment or repayment with one or both hh applying pressure; VI = no repayment or repayment with no pressure or one hh applying pressure; VII = repayment with no pressure or one hh applying pressure; VIII = no repayment or repayment with no pressure or one or both hh applying pressure.

Figure 2: $\phi(r)+\gamma > \phi(r)+\gamma-\eta > \phi(r) > \gamma > \phi(r)-\eta$



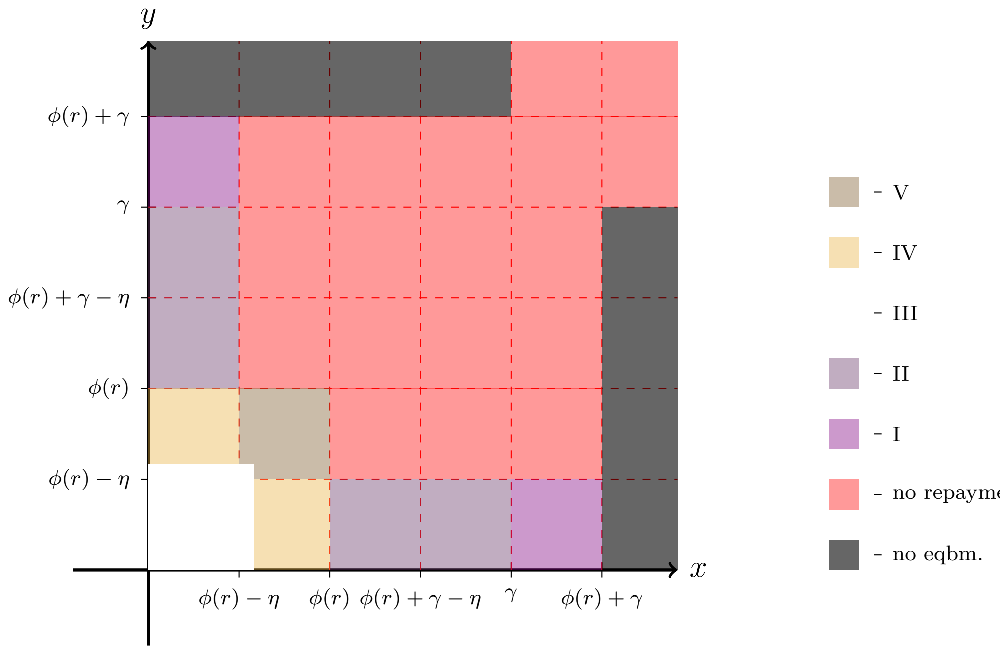
I = no repayment or repayment with one hh applying pressure; II = no repayment or repayment with both hh applying pressure; III = no repayment or repayment with no pressure or both hh applying pressure; IV = repayment with one or both hh applying pressure; V = no repayment or repayment with no pressure; VI = no repayment or repayment with no pressure or one hh applying pressure; VII = repayment with no pressure or one hh applying pressure; VIII = repayment with no pressure or one hh applying pressure.

Figure 3: $\phi(r) + \gamma > \phi(r) + \gamma - \eta > \gamma > \phi(r) > \phi(r) - \eta$



I = no repayment or repayment with one hh applying pressure; II = no repayment or repayment with both hh applying pressure; III = repayment with both hh applying pressure; IV = repayment with one or both hh applying pressure; V = no repayment or repayment with no pressure; VI = repayment with one hh applying pressure; VII = repayment with no pressure or one hh applying pressure; VIII = repayment with no pressure or one hh applying pressure.

Figure 4: $\phi(r) + \gamma > \gamma > \phi(r) + \gamma - \eta > \phi(r) > \phi(r) - \eta$



I = no repayment or repayment with one hh applying pressure; II = repayment with one hh applying pressure; III = not feasible since $X_i > 0$; IV = repayment with no pressure or one hh applying pressure; V = no repayment or repayment with no pressure.

As illustrated above, for any sub-game originating at a given state of the world, λ, existence of *admissible equilibrium* is not guaranteed. It can be verified using the conditions enlisted in the appendix that the following assumption guarantees existence of *admissible* *equilibria.*

Assumption 2 *Condition to guarantee existence of admissible equilibrium.* For any $i \in \{A, B\}$ and $j \in \{A, B\} \setminus \{i\}$, both

$$\max\{\phi(r) + \gamma, \gamma\} < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big)$$

and

$$\gamma > u\big(w + R\lambda_j\big) - u\big(w + R\lambda_j - r\tilde{x}_j(\lambda)\big)$$

are not simultaneously true in any λ.

4 Full game

4.1 Setup

I now enrich the repayment game with important features commonly observed in group lending programs. In this paper, I incorporate the following features:

1. Households can choose between high risk high reward (*risky*) and low risk low reward (*safe*) projects.
2. Endogenize the interest rate that the MFI sets.
3. MFIs invest resources in building relationships with clients. Further, MFIs often leverage this relationship to discourage strategic defaults (*institutional pressure*).

The *full game* I prescribe begins by the *not for profit* MFI first choosing whether or not to make its loan available. Denote this decision by $O \in \{0, 1\}$, where the MFI choosing to *operate* (make its loan available) is assigned $O = 1$. If operating, the MFI can raise capital at rate c and must chose the interest rate r at which the loan is offered. Having observed the interest rate set by the MFI, both households in the group simultaneously choose between the *safe* and *risky* projects. Denote these choices by $L_i \in \{L_{risky}, L_{safe}\}$ for $i \in \{A, B\}$. Based on the interest rate r, projects chosen L_A, L_B, the repayment game (described above) is played.

Here, it is convenient to elaborate on the two types of lotteries. The stochastic returns are governed by probability laws:

$$\text{return in } L_{safe} = \begin{cases} R_i, & \text{with probability } \mu \\ \frac{R_i}{2}, & \text{with probability } 1 - \mu \end{cases}$$

$$\text{return in } L_{risky} = \begin{cases} 2R_i, & \text{with probability } \mu \\ 0, & \text{with probability } 1 - \mu \end{cases}$$

A crucial feature of these projects is that L_{risky} has a higher excepted return and higher variance than L_{safe}. This is in contrast to the setup in Stiglitz (1990) where the risky project has both lower mean and higher variance. Since participants now choose between projects, the state space (of investment outcomes) has expanded, i.e. $\lambda \in \{0, 0.5, 1, 2\}^2$.

Defined below is the new social repayment norm that accounts for the different levels of outcomes that may be realized.

$$\tilde{x}_i(\lambda) = \begin{cases} \theta, \text{ if } \text{ if } \lambda_i = \lambda_j = 0 \\ \frac{\theta}{\lambda_j}, \text{ if } \lambda_i = 0, \ \lambda_j \neq 0 \\ 1 + (1 - \frac{\theta}{\lambda_i}), \text{ if } \lambda_i \neq 0, \ \lambda_j = 0 \\ 1, \text{ if } \lambda_i \neq 0, \ \lambda_j \neq 0 \end{cases}$$

where $j = \{A, B\} \setminus \{i\}$. In this specification, the norm takes into account the outcome of the successful investment realization and proposes larger co-payment amounts in states with higher outcomes.

Definition 5 A (pure) strategy profile of the *full game* $\Sigma = \Big(O, r, \big(L_A(r), L_B(r), \sigma(r, L_A, L_B)_{\forall L_A, L_B}\big)_{\forall r}\Big)$ lists the decision to enter, the interest rate chosen by the MFI, the projects chosen and the strategy of pressure and repayments by group members at each interest rate that may be set by the MFI, i.e. for all $L_A, L_B \in \{L_{safe}, L_{risky}\}$, $\sigma(r, L_A, L_B) = \Big\{\delta_A^\lambda(r, L_A, L_B), \delta_B^\lambda(r, L_A, L_B), d^\lambda(r, L_A, L_B) : \forall \lambda \in \{0, 0.5, 1, 2\}^2\Big\}$.

Definition 6 The *endogenous investment distribution* for any strategy profile $\Sigma = \Big(O, r, \big(L_A(r), L_B(r), \sigma(r, L_A, L_B)_{\forall L_A, L_B}\big)_{\forall r}\Big)$ is defined as $\mathbb{P}(\lambda|\Sigma) = \mathbb{P}(\lambda|L_A(r), L_B(r))$. This is detailed in the table below:

Table 1: $\mathbb{P}(\lambda|L_A, L_B)$

λ	$\mathbb{P}(\lambda\|L_{safe}, L_{safe})$	λ	$\mathbb{P}(\lambda\|L_{safe}, L_{risky})$	λ	$\mathbb{P}(\lambda\|L_{risky}, L_{risky})$
(0.5,0.5)	$(1-\mu)^2$	(0.5,0)	$(1-\mu)^2$	(0,0)	$(1-\mu)^2$
(0.5,1)	$\mu(1-\mu)$	(0.5,2)	$\mu(1-\mu)$	(0,2)	$\mu(1-\mu)$
(1,0.5)	$\mu(1-\mu)$	(1,0)	$\mu(1-\mu)$	(2,0)	$\mu(1-\mu)$
(1,1)	μ^2	(1,2)	μ^2	(2,2)	μ^2

Definition 7 $\pi(\Sigma)$ denotes the *default rate* associated with any strategy profile Σ. i.e. $\pi(\Sigma) = \sum_\lambda \mathbb{P}(\lambda|\Sigma) 1\big\{d_A^\lambda(\Sigma)\tilde{x}_A(\lambda) + d_B^\lambda(\Sigma)\tilde{x}_B(\lambda) < 2\big\}$.

Assumption 3 *Allowed interest rates.* Restrict interest rates that the MFI may set (i.e. restriction on the action space) to:

$$r \in \left\{\frac{c}{\mu^2}, \frac{c}{\mu^2 + \mu(1-\mu)}, \frac{c}{\mu^2 + 2\mu(1-\mu)}, c\right\}$$

These rates correspond to those values at which the MFI breaks even (earns zero profit on expectation) when repayment occurs:

- if both households succeed (i.e. $\lambda_i \geq 1, \lambda_j \geq 1$),
- if both households succeed or A succeeds while B fails but not vice versa,

- if at least one household succeeds,
- always

respectively. This assumption will be without loss in generality (in equilibrium) when paired with the following assumption.

Assumption 4 A strategy profile satisfies *monotone repayment maximality* if the following properties hold:

1. $\forall r$, if full repayment occurs in state λ, then full repayment also occurs in all $\lambda' \geq \lambda$ where every group member's investment outcome is at least as successful;
2. $\forall \lambda$, if full repayment occurs at interest rate r, then full repayment also occurs at any lower interest rate $r' < r$.

The payoffs to the households remain unchanged and require a notational update to account for the choice of investment projects. For any $i \in \{A, B\}$,

$$U_i\Big(\Sigma\Big) = O \times \sum_{\lambda} \mathbb{P}(\lambda|\Sigma) \times U_i\big(\sigma(r, L_A, L_B); \lambda\big) + (1 - O) \times u(w)$$

The payoff to the MFI associated with strategy Σ is represented as:

$$U_{MFI}(\Sigma) = 1\Big\{r_\Sigma \cdot \pi\big(\Sigma\big) \geq c\Big\} \cdot O + 0.1(1 - O)$$

where r_Σ is the interest rate suggested in the strategy profile, Σ. The intuition is that if the MFI is able to recover the cost of providing loans (in expectation), it gets a utility of 1. To the contrary, if it unable to do so, the utility of the MFI has been normalized to 0. If the MFI chooses not to operate it receives 0.1 utils, this implies that the MFI would rather choose not to operate if it cannot recover costs.

4.2 Continuation value

The notion of the continuation value defined earlier as the net-present value of having continued access to this source of credit needs to be updated to reflect the choice of projects. In the full game, this is denoted as $\phi(r, L_A, L_B)$ to indicate that this value is sensitive to the choice of interest rate as well as the projects, but *not* the choices in the repayment game. Consequently, this allows households to value loans offered at a higher interest rates less than those offered at a lower interest rate. This will in turn affect their repayment decisions. Furthermore, this representation allows households to internalize how members of their group use their loans and the net-present value is suitable adjusted. This also affects repayment decisions. Ideally, the repayment decisions would also be internalized to yield an analogue of the stationary equilibrium of repeated games. Instead, this is restricted in favor of tractability. I make this specific assumption with regard to repayment decisions being factored into the continuation value:

Assumption 5 For the purposes of computing the continuation value, at any interest rate r and project (investment) choice L_A, L_B, full repayment is assumed to occur whenever feasible, i.e. full repayment occurs in all state λ such that $\tilde{x}_A(\lambda) + \tilde{x}_B(\lambda) = 2$.

Thus, given the *repayment norm* detailed above, repayment is assumed to occur in all states expect $\lambda = (0, 0)$. Please refer to the appendix for details on how $\phi(\cdot, \cdot, \cdot)$ is set.

4.3 Equilibrium

Definition 8 *Monotone repayment equilibrium in the full game.*
A strategy profile $\Sigma = \Big(O, r, \big(L_A(r), L_B(r), \sigma(r, L_A, L_B)_{\forall L_A, L_B}\big)_{\forall r}\Big)$ is an admissible equilibrium if the following conditions hold:

1. $\forall r, L_A, L_B,\ \sigma(r, L_A, L_B)$ is itself an *admissible equilibrium* in the repayment game,
2. $\forall r,\ L_A(r)$ and $L_B(r)$ are simultaneous best responses given the strategy in the group repayment game, $\sigma(r, \cdot, \cdot)$,
3. O, r chosen to satisfy

$$O = \begin{cases} 1, & \text{if } r \cdot \pi\big(\Sigma\big) \geq c \\ 0, & \text{otherwise} \end{cases}$$

Existence of *monotone repayment equilibria* in the full game is closely tied with the existence of *admissible equilibria* in the repayment game. It is required that for each r, L_A, L_B, there exists an admissible equilibrium in the repayment game. Thus, $\forall r,\ \forall \lambda \in \{0, 1/2, 1, 2\}^2$, the conditions in assumption 2 are not simultaneously satisfied. Further, at any interest rate r, a simultaneous best response in choice of projects (investments) must exist. Here, we can exploit the symmetry of the game to ensure existence. Note, that in the sub-game where households chose projects, the payoffs may be summarized as follows for symmetric profiles: where x, y, z, w correspond to payoffs associated with $\sigma(r, L_{safe}, L_{safe}), \sigma(r, L_{safe}, L_{risky})$ and

Table 2: Normal form representation of project selection stage game

		HH 2: Safe	HH 2: Risky
HH 1	Safe	x, x	w, z
HH 1	Risky	z, w	y, y

$\sigma(r, L_{risky}, L_{risky})$ repayment strategy profiles. It is well know that pure strategy equilibrium always exist in such games.

4.4 Selection Mechanism

I now turn my attention to the role of *institutional pressure*. As noted earlier, it is well documented that MFIs invest resources in building relationships with their clients and often leverage this relation to discourage strategic defaults among groups (see Haldar, Stiglitz (2016)). Further, early iterations of microfinance programs (the focus of this paper) preferred keeping interest rates low to ensure accessibility. Consider the following quote from then acting Managing Director of the Grameen MFI, Ratan Kumar Nag "*We are now charging the highest 20.0 per cent interest against a loan though the ceiling of interest is 27.0 per* *cent fixed by the Microcredit Regulatory Authority (MRA).*" He also suggested that the loan recovery rate was 99.05% in 2016. In the model, this will be incorporated by means of a *selection mechanism.*

Assumption 5 In the event of multiplicity of equilibria, the application fo *institutional pressure* ensures that the market *selects* the equilibrium with the high pressure and repayment (in each relevant sub-game). Call these *maximal monotone repayment equilibrium.*

It is useful to point out that maximal equilibrium is not unique. For example, a given repayment profile might be sustained by pressure by either household individually. However, it does ensure enough tractability to conduct welfare analysis.

By way of anecdotal evidence, consider the case of Andhra Pradesh, India. Initially, the market was largely controlled by SKS, which was built in the image of the early iteration of the Grameen bank. By late 2000s, the industry saw an influx of large number of MFIs followed by rapid commercialization. As Ballem et al from Microsave (then a competitor to SKS) recount: "*Most MFIs are mono-service credit companies providing standard basic joint liability group (JLG) loans to customers. There has been only a limited focus on clients; be it in terms of assessing their capacity to repay or in developing appropriate products to suit their needs. Microsave has often observed that despite the MFI management's protestation to the contrary, most clients see MFIs as just another source of credit, rather than institutions interested in client welfare. The rapid influx of capital resulted in rapid expansion in scale without adequate investment in building customer relationships.*" This rapid commercialization was immediately followed by a collapse of the industry to the point of government intervention. Repayment rates plummeted to 10 - 15% while interest rates soared (as suggested by Ballem and colleagues). This is suggestive of the role of the relationship between MFIs and its clients in ensuring high repayment and consequently low interest rates.

5 Welfare Analysis

The objective of this paper is to study the inefficiencies arising from top-down pressure (*institutional pressure* as well as *peer pressure*) in group lending programs. To that end, I now describe the problem faced by a utilitarian planner in this environment. The planner seeks to maximize the expected household welfare subject to meeting the zero-profit condition of the MFI. Define $h(\cdot) : \{0, 1/2, 1, 2\}^2 \to \{0, 1\}$ as the (full) repayment decision of the planner in each state of the world[2]. Consequently, the planner's problem maybe expressed as follows:

$$\max_{O,r,h(.),L_A,L_B} \; (1-O)2u(w) +$$
$$O\big[\mathbb{E}U_{A,\lambda|L_A,L_B}(r,h(.)) + \mathbb{E}U_{B,\lambda|L_A,L_B}(r,h(.))\big] \cdot 1\Big\{r \cdot \mathbb{E}_{\lambda|L_A,L_B}\big[h(\lambda) \cdot 1\{\tilde{x}_A(\lambda) + \tilde{x}_B(\lambda) = 2\}\big] \geq c\Big\}$$

[2] Not only would the planner never need to apply pressure to induce repayment, but also, applying pressure is costly ($\eta > 0$). Hence, we suppress this decision and stipulate that the planner never applies pressure.

where

$$\mathbb{E}U_{i,\lambda|L_A,L_B}(r,h(.)) = \sum_{\lambda} \mathbb{P}(\lambda; L_A, L_B)\big[h(\lambda)\cdot U_i(\delta^{\lambda}=(0,0), d^{\lambda}=(1,1);\lambda) + (1-h(\lambda))\cdot U_i(\delta^{\lambda}=(0,0), d^{\lambda}=(0,0);\lambda)\big]$$

where the actions of the planner are such that interest rate $r \in \left\{\frac{c}{\mu^2}, \frac{c}{\mu^2+\mu(1-\mu)}, \frac{c}{\mu^2+2\mu(1-\mu)}, c\right\}$, the choice of lotteries $L_A, L_B \in \{L_1, L_2\}$ and repayment decisions $h : \{0, 1/2, 1, 2\}^2 \to \{0, 1\}$ such that when $\mathbb{P}(\lambda; L_A, L_B = 0)$ then $h(\lambda) = 0$. Owing to the finiteness of the problem, a solution exists.

6 Discussion

I now discuss some of the intuition generated by the model. Since characterizing the set of equilibrium in such game in terms of the parameters is unyielding, I present numerical computations of the *maximal monotone repayment equilibrium* as well as the *planner's problem* for numerous parameter values. I then provide intuition for the mechanics at work within the model that drive the observed results. These results are detailed in Table 3.

Table 3: Main Results

	Game		Planner	
Parameter	Repayment	Project	Repayment	Project
μ_H, α_H	always repay	safe projects	always repay	risky projects
μ_H, α_L	repay unless both fail	risky projects	always repay	risky projects
μ_L, α_H	repay unless both fail	risky projects	always repay	risky projects
μ_L, α_L	always repay	safe projects	always repay	risky projects

$\mu_H = 0.95, \mu_L = 0.65, \alpha_L = 1, \alpha_H = 3, w = 5, R = 3, \theta = 0.25, \gamma = u_{(\alpha=1)}(S), \eta = \gamma/100.$

In all the numerical examples considered, it is *efficient* for both households to invest in the *risky* lottery and repay whenever feasible (i.e. in all states, except $\lambda = (0,0)$). The first numerical example explores the case where both the probability of success of the projects (i.e. $P[\lambda_i \geq 1]$) as well as the household risk aversion are *high*. Here, in equilibrium both households choose the *safe* project and repay in every state of the world. It is also observed that this repayment is sustained *without peer pressure* from either household in any state of the world[3]. The repayment is sustained by a *high continuation value* owing to the high probability of success. The conservative project choice is a consequence of *high risk aversion*. This ensures that the continuation value of choosing the risky projects is *lower* than the same safe lotteries. This is illustrated in the *Figure 6* below. The second example

[3] Note that given the selection mechanism proposed, *institutional pressure* is always applied. It is worth noting, there also exists a *monotone repayment equilibrium* to this game where no repayment occurs in any state and consequently, the MFI chooses *not to operate*. This establishes that in this game, *peer pressure* is not sufficient to ensure repayment.

maintains a high probability of success but sets the household risk aversion level *low*. Here, in equilibrium both households choose the *risky* project and repay in every state of the world. Repayment occurs in all states except when both households fail simultaneously (i.e. $\lambda = (0, 0)$). Repayment (when it occurs) is sustained by peer pressure from *both households*. Project choices are a result of the fact that households are only mildly risk averse. It is evident from *Figure 6* below that the continuation value of choosing the risky projects is now *higher* than the same for safe projects. The third and fourth examples set a low probability of success. The striking finding in these examples is that, contrary to the earlier examples, with *high risk aversion*, households choose the *risky* projects and *repay whenever feasible*. While with *low risk aversion*, households choose the *safe* projects and *repay always*. This switch is a consequence of the switch in the continuation values as illustrated in *Figure 7*. When the probability of success is low, continuation values are *lower*. Thus, repayment of highly risk averse households is sustained through *peer pressure from both households* in all states. In equilibrium, risk averse households internalize the excessive pressure they will come under regardless of the projects chosen. In the safe project, repayment occurs even in the state where both households fail simultaneously (an event that now occurs with higher probability). However, if a household invests in the risky project instead, it has to repay a smaller share if it fails. Thus, both households rationally respond by choosing the risky project.

Figure 6: Continuation value as a function of α when $\mu = \mu_H$

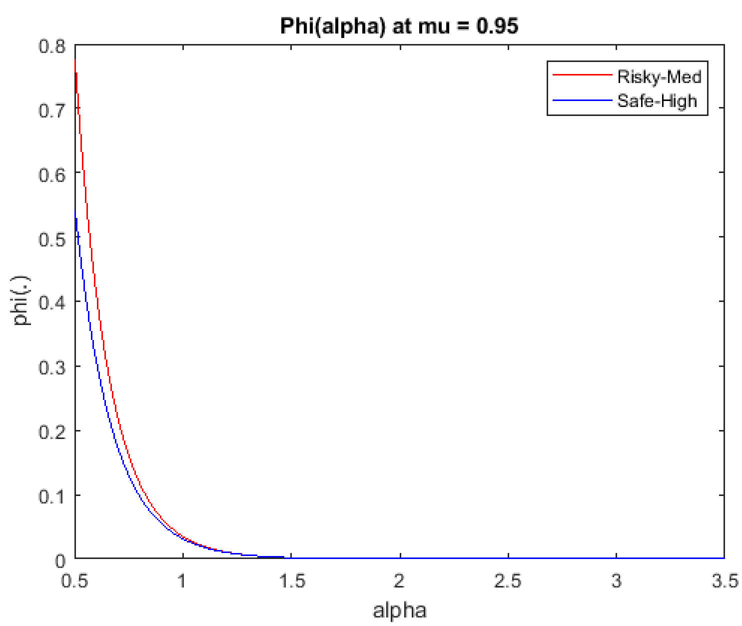
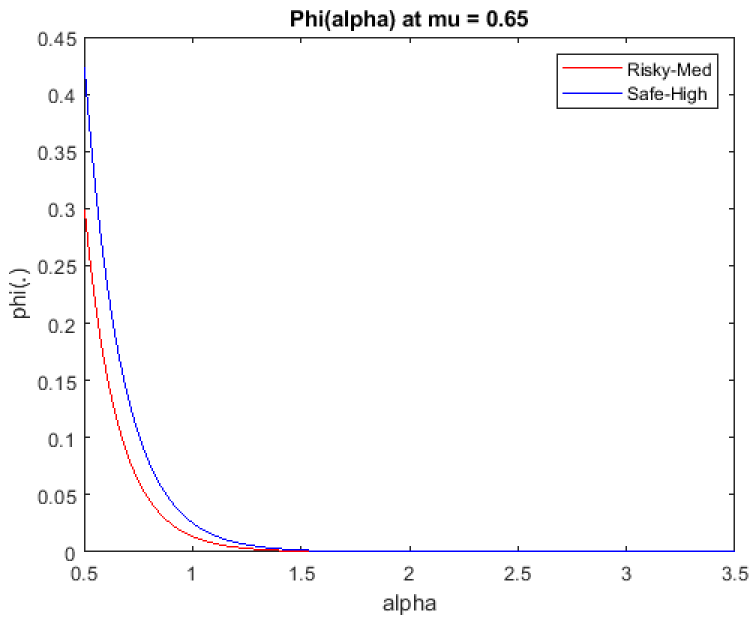


Figure 7: Continuation value as a function of α when $\mu = \mu_L$



It is interesting to note that this pattern is robust to changes in the initial wealth level (w). This suggests a generalizability of the finding that over-exertion of pressure on debtors to repay leads to an inefficient allocation of capital. This is consistent with findings in Acharya, Amihud and Litov (2011) and Kind, Wende (2019). Another useful generalization is to allow for the project returns to be correlated across households. In an attempt to study this, the returns to the projects are re-parametrized as in *Table 4*.

Table 4: $\mathbb{P}(\lambda|L_A, L_B)$ with correlation parameter ρ

λ	$\mathbb{P}(\lambda\|L_{safe}, L_{safe})$	λ	$\mathbb{P}(\lambda\|L_{safe}, L_{risky})$	λ	$\mathbb{P}(\lambda\|L_{risky}, L_{risky})$
(0.5,0.5)	$(1-\mu)^2 + \rho\mu(1-\mu)$	(0.5,0)	$(1-\mu)^2 + \rho\mu(1-\mu)$	(0,0)	$(1-\mu)^2 + \rho\mu(1-\mu)$
(0.5,1)	$\mu(1-\mu)(1-\rho)$	(0.5,2)	$\mu(1-\mu)(1-\rho)$	(0,2)	$\mu(1-\mu)(1-\rho)$
(1,0.5)	$\mu(1-\mu)(1-\rho)$	(1,0)	$\mu(1-\mu)(1-\rho)$	(2,0)	$\mu(1-\mu)(1-\rho)$
(1,1)	$\mu^2 + \rho\mu(1-\mu)$	(1,2)	$\mu^2 + \rho\mu(1-\mu)$	(2,2)	$\mu^2 + \rho\mu(1-\mu)$

For low values of correlation (ρ) the results are identical to those above. However, an interesting features occur for high correlation. In *Table 5*, I present the results for the extreme case of fully correlated projects. Here, the interesting finding is for the case with high probability of success and only mildly risk averse households. One household chooses the *risky* project while the other chooses the *safe* project. Repayment occurs in all states and is sustained by peer pressure from both households. Suppose household A chose the safe project, note that household B knows that choosing the risky lottery has no downside, since either both fail or both succeed. If both fail, the peer pressure sustains repayment. In response to household B choosing the risky project, household A does not also choose the risky lottery. This is because if both fail, there will be no repayment and consequently the continuation value is lower.

Table 5: Results with fully correlated projects

	Game		Planner	
Parameter	Repayment	Project	Repayment	Project
μ_H, α_H	always repay	safe projects	always repay	risky projects
μ_H, α_L	always risky	mixed projects	always repay	risky projects
μ_L, α_H	repay unless both fail	risky projects	always repay	risky projects
μ_L, α_L	always repay	safe projects	always repay	risky projects

$\mu_H = 0.95, \mu_L = 0.65, \alpha_L = 1, \alpha_H = 3, w = 5, R = 3, \theta = 0.25, \gamma = u_{(\alpha=1)}(S), \eta = \gamma/100.$

These results suggest the detrimental role of *excessive top-down pressure* in group lending programs. Excessive pressure to repay induces the high rates of repayment observed among *not-for profit* MFI. This is often celebrated as an indicator of the success of group lending programs in solving problems of moral hazard and adverse selection associated with lending in the absence of financial collateral. This paper demonstrates how this transfer of risk from the MFI to the households result in the latter inefficiently choosing the invest the loan in safer projects rather than high risk high reward projects which could substantially aid the alleviation of poverty.

References

[1] Viral V. Acharya, Yakov Amihud, and Lubomir Litov. Creditor rights and corporate risk-taking. *Journal of Financial Economics*, 102(1):150–166, 2011.

[2] Christian Ahlin. The role of group size in group lending. *Journal of Development Economics*, 115:140–155, 2015.

[3] Christian Ahlin and Matthew Suandi. A Matter of Experience? Understanding the Decline in Group Lending. *Oxford Bulletin of Economics and Statistics*, 2019.

[4] Christian Ahlin and Robert M. Townsend. Selection into and across credit contracts: Theory and field research. *Journal of Econometrics*, 136(2):665–698, 2007.

[5] Manuela Angelucci, Dean Karlan, and Jonathan Zinman. Microcredit Impacts: Evidence from a Randomized Microcredit Program Placement Experiment by Compartamos Banco . *American Economic Journal: Applied Economics*, 7(1):151–182, 2015.

[6] Richard Arnott and Joseph E Stiglitz. Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out of Peer Monitoring? *The American Economic Review*, 81(1):179–190, 1991.

[7] Orazio Attanasio, Britta Augsburg, Ralph De Haas, Emla Fitzsimons, and Heike Harmgart. Group Lending or Individual Lending? Evidence from a Randomized Field Experiment in Rural Mongolia. 2013.

[8] Orazio Attanasio, Britta Augsburg, Ralph De Haas, Emla Fitzsimons, Heike Harmgart, Erik Berglöf, Robert Lensink, David Mckenzie, Gabriel Natividad, David Roodman, Marta Serra Garcia, Jeromin Zettelmeyer, Maria Lotito, Bold Magvan, Norov Sanjaajamts, Benjamin Shell, Oksana Pak, Erin Burgess, Stephen Butler, Pamela Loose, and Jeffrey Telgarsky. The Impacts of Microfinance: Evidence from Joint-Liability Lending in Mongolia . *American Economic Journal: Applied Economics*, 7(1):90–122, 2015.

[9] Timothy Besley and Stephen Coate. Group lending, repayment incentives and social collateral. *Journal of Development Economics*, 46(1):1–18, 1995.

[10] By Francis Bloch, Garance Genicot, and Debraj Ray. Reciprocity in Groups and the Limits to Social Capital Reciprocity. *The American Economic Review: Papers and Proceedings*, 97(2):65–69, 2007.

[11] Bruno Crépon, Florencia Devoto, Esther Duflo, and William Parienté. Estimating the Impact of Microcredit on Those Who Take It Up: Evidence from a Randomized Experiment in Morocco . *American Economic Journal: Applied Economics*, 7(1):123–150, 2006.

[12] Jonathan de Quidt, Thiemo Fetzer, and Maitreesh Ghatak. Commercialization and the decline of joint liability microcredit. *Journal of Development Economics*, 134(March 2017):209–225, 2018.

[13] Jonathan De Quidt and Maitreesh Ghatak. Is the Credit Worth It? for-Profit Lenders in Microfinance With Rational and Behavioral Borrowers. *Annals of Public and Cooperative Economics*, 89(1):175–199, 2018.

[14] Maitreesh Ghatak. Group lending, local information and peer selection. *Journal of Development Economics*, 60(1):27–50, 1999.

[15] Maitreesh Ghatak. Screening by the company you keep: Joint liability lending and the peer selection. *Economic Journal*, 110(465):601–631, 2000.

[16] Maitreesh Ghatak and Timothy W. Guinnane. The economics of lending with joint liability: Theory and practice. *Journal of Development Economics*, 60(1):195–228, 1999.

[17] Rachel Glennerster, Abhijit Banerjee, Ester Duflo, and Cynthia Kinnan. The miracle of microfinance? Evidence from a randomized evaluation. *American Economic Journal: Applied Economics*, 7(1):22–53, 2015.

[18] Antara Haldar and Joseph E. Stiglitz. Group lending, joint liability, and social capital: Insights from the Indian microfinance crisis. *Politics and Society*, 44(4):459–497, 2016.

[19] Axel Kind and Sabine Wende. The Ex Ante Effect of Creditor Rights on Corporate Financial and Investment Policy*. 2019.

[20] Malgosia Madajewicz. Joint liability versus individual liability in credit contracts. *Journal of Economic Behavior and Organization*, 77(2):107–123, 2011.

[21] Richard Montgomery. Disciplining or protecting the poor? Avoiding the social costs of peer pressure in micro-credit schemes. *Journal of International Development*, 8(2):289–305, 1996.

[22] Mark M Pitt and Shahidur R Khandker. The Impact of Group-Based Credit Programs on Poor Households in Bangladesh: Does the Gender of Participants Matter? Mark. *Journal of Political Economy*, 106(5):20, 1998.

[23] David Roodman and Jonathan Morduch. Journal of Development Studies The Impact of Microcredit on the Poor in Bangladesh: Revisiting the Evidence. *Journal of Development Studies*, 50(4):583–604, 2013.

[24] Caroline E. Schuster. *Social collateral : women and microfinance in Paraguay's smuggling economy.*

[25] Joseph E. Stiglitz. Peer monitoring and credit markets. *World Bank Economic Review*, 4(3):351–366, 1990.

[26] Robert M. Townsend and Christian Ahlin. Using Repayment Data to Test Across Theories of Joint Liability Lending. *The Economic Journal*, 117(February), 2003.

[27] Hal Varian. Monitoring agents with other agents. *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift Für Die Gesamte Staatswissenschaft*, 146(1):153–174, 1990.

Appendix

Proof for Proposition 1:

Suppose to the contrary that the repayment is not full, then, $\sum_j d^\lambda(\delta^\lambda)_j \tilde{x}_j(\lambda) < 2$. Thus, both households in the group are penalized and are unable to participate in the program thereafter. Let i denote a household such that $\delta_i^\lambda = 1$. Exploring the profitability of a one shot deviation for node i. Consider the utility to household i of applying pressure ($j = \{A, B\} \setminus i$):

$$\begin{aligned} U_i(\text{ applying pressure }, \sigma; \lambda) = u(w + R\lambda_i - r\tilde{x}_i(\lambda)d_i^\lambda) - \eta \\ - \gamma \max\left\{(1 - d_j^\lambda), \delta_j^\lambda(1 - d_i^\lambda)\right\} \end{aligned}$$

Consider now its utility of not applying pressure:

$$\begin{aligned} U_i(\text{ no pressure }, \sigma; \lambda) = u(w + R\lambda_i - r\tilde{x}_i(\lambda)d_i^\lambda) \\ - \gamma\delta_j^\lambda(1 - d_i^\lambda) \end{aligned}$$

Given $\eta > 0$, one can see that

$$U_i(\text{ no pressure }, \sigma; \lambda) > U_i(\text{ applying pressure }, \sigma; \lambda)$$

Hence there exists a profitable one-shot deviation for i implying that σ cannot be a sub-game perfect equilibrium so long as $\mathbb{P}(\lambda) > 0$. ■

Illustration for monotone repayment response to peer pressure assumption:

Consider the following behavior that would be consistent with sub-game perfection driven entirely by the complementarity of repayments:

In state $\lambda \neq (0, 0)$:

$$\left\{ \delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (1,1), & \text{if } \delta = (0,0) \\ (0,0), & \text{if } \delta \neq (0,0) \end{cases} \right\}$$

Although this is optimal, as long as

$$\begin{aligned} \phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma \\ \phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma \end{aligned}$$

While the model would regard this behavior as rational, I would like to rule out this kind of behavior. That is, if the participants select an equilibrium with full repayment without pressure, then the addition of pressure should not cause them to select an equilibrium with lower repayment. ▲

Proof for Proposition 2:

When $\lambda = (0,0)$, notice that $\tilde{x}_A(\lambda) = \tilde{x}_B(\lambda) = 0$, i.e. the social repayment norm does not yield full repayment. In the parlance of the model, $\phi(r) \cdot 1\{d_A\tilde{x}_A(\lambda) + d_B\tilde{x}_B(\lambda) = 2\} = 0$. Thus, there is no incentive for households to apply any costly pressure ($\eta > 0$) to force other households to follow the social repayment norm. An application of the contrapositive of the *proposition 4* yields that in any sub-game perfect equilibrium, no pressure is applied, the repayment and punishments observed are identical. ■

Proof for Proposition 3:

1. Suppose to the contrary that $d^\lambda(0,0) = (1,0)$ in some state of the world λ. Looking at the payoffs to household A implied by the game.
   When household A repays:

   $$U_A(\text{repay}) = u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) \cdot 0$$

   When household A deviates by not repaying:

   $$U_A(\text{deviate}) = u\big(w + R\lambda_A\big) + \phi(r) \cdot 0 - \gamma \cdot 0$$

   Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. This yields a contradiction in every λ. By a symmetric argument, one can establish a contradiction for $d^\lambda(0,0) = (0,1)$.
   So, this establish that for all $\lambda \neq (0,0)$, $d^\lambda(0,0) \in \{(0,0),\ (1,1)\}$.
   This reduces the number of candidate sub game optimal repayment plays to 128.
2. Suppose in some state $\lambda \neq (0,0)$ where households play a sub-game perfect repayment play with $d^\lambda(.)$. Now, consider the event where household A applies pressure; notice that for household A, utility of following social repayment norm is

   $$U_A(\text{repay}) = u\big(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)-\gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)-\gamma$$

   while the utility of deviating is

   $$U_A(\text{deviate}) = u\big(w + R\lambda_A\big) - \gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w + R\lambda_A\big) - \gamma$$

   Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. Thus, $d^\lambda(1,0) = (1,0)$ in not optimal.
   By a symmetric argument, one can establish a contradiction for $d^\lambda(0,1) = (0,1)$ is sub optimal in any $\lambda \neq (0,0)$.
   This further reduces the number of candidate sub game optimal repayment plays to 64.
3. Suppose in some state $\lambda \neq (0,0)$ where households play a sub-game perfect equilibrium with repayment play $d^\lambda(1,1) = (1,0)$. Now, consider the event where household A applies pressure, notice that for household A, utility of following social repayment norm is

   $$U_A(\text{repay}) = u\big(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)-\gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)-\gamma$$

while the utility of deviating is

$$U_A(\text{deviate}) = u\big(w + R\lambda_A\big) - \gamma \max\left\{\delta_A(1-d_B), \delta_B(1-d_A)\right\} = u\big(w + R\lambda_A\big) - \gamma$$

Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. Thus, $d^\lambda(1,1) = (1,0)$ in not optimal.
By a symmetric argument, one can establish a contradiction for $d^\lambda(1,1) = (0,1)$ is sub optimal in any $\lambda \neq (0,0)$.
This further reduces the number of candidate sub game optimal repayment plays to 36.
■

List of 11 repayment plays for Proposition 4:

- When $\lambda \neq (0,0)$:
  1. $$d^\lambda(\delta) = (0,0), \text{ for all } \delta$$
  2. $$d^\lambda(\delta) = (1,1), \text{ for all } \delta$$
  3. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$$
  4. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$$
  5. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B < 2 \\ (1,1), \text{ if } \delta_A + \delta_B = 2 \end{cases}$$
  6. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0 \\ (0,1), \text{ if } \delta_A = 1, \ \delta_B = 0 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$
  7. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1, \ \delta_B = 1 \end{cases}$$
  8. $$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$$

9.

$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \ \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$$

10.

$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B = 0 \\ (1,1), \text{ if } \delta_A + \delta_B \geq 1 \end{cases}$$

11.

$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$

- When $\lambda = (0,0)$, restrict attention to $d^{\lambda}(\delta) = (0,0), \ \forall \delta$. ▲

Characterizing the admissible equilibria:

Following the arguments of backward induction:
Stage 2 best responses
Now, restricting attention to these 11 repayment plays, consider the set of sub-game perfect equilibria that emerge and how that varies over the parameter space. Given the assumption that deals with optimal play in sub-games with $\lambda = (0,0)$, the rest of the analysis of the equilibrium will focus on dealing with sub-games where $\lambda \neq (0,0)$. For households $i \in \{A, B\}$ and $j \in \{A, B\} - i$, let $d_i(\delta_i, \delta_j, d_j)$ denote the best response by household i given δ_i, δ_j and d_j which also depends on λ but has been suppressed in notation to enhance readability:

1. $d_i(\delta_j = d_j = 0) =?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) - \gamma\delta_i - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma\delta_i - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \quad &< \quad u\big(w + R\lambda_i\big) \end{aligned}$$

since $r\tilde{x}_i(\lambda) > 0$ for all $\lambda \neq (0,0)$. Thus,

$$d_i(\delta_j = d_j = 0) = 0, \text{ for all parameter values}$$

2. $d_i(\delta_j = 0, \ d_j = 1) =?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) \end{aligned}$$

Thus,

$$d_i(\delta_j = 0, \ d_j = 1) = \begin{cases} 1, \text{ if } \phi(r) \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, \text{ if } \phi(r) < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$$

3. $d_i(\delta_j = 1,\ d_j = 0) = ?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) - \gamma\delta_i - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma - \eta\delta_i \\ (1-\delta_i)\gamma \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{aligned}$$

This depends on whether or not node i is applying pressure and thus can be split into two cases:

(a) $d_i(\delta_i = 0,\ \delta_j = 1,\ d_j = 0) = \begin{cases} 1, & \text{if } \gamma \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, & \text{if } \gamma < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$

(b) $d_i(\delta_i = 1,\ \delta_j = 1,\ d_j = 0) = 0,$ for all parameter values

4. $d_i(\delta_j = 1,\ d_j = 1) = ?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma \\ \gamma + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{aligned}$$

Thus,

$$d_i(\delta_j = 1,\ d_j = 1) = \begin{cases} 1, & \text{if } \gamma + \phi(r) \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, & \text{if } \gamma + \phi(r) < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$$

This is now used to find parametric ranges where each of the 11 repayment plays are supported for different realizations of $\lambda \neq (0,0)$.

1.

$$d^\lambda(\delta) = (0,0), \text{ for all } \delta$$

This is supported when:

$$\begin{aligned} \gamma &< u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \\ \gamma &< u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

2.

$$d^\lambda(\delta) = (1,1), \text{ for all } \delta$$

This is supported when:

$$\begin{aligned} \phi(r) &\geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \\ \phi(r) &\geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

3.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$$

This is supported when:

$$\begin{aligned} \phi(r) + \gamma &\geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \\ \phi(r) &\geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma \end{aligned}$$

4.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}$$

This is supported when:

$$\phi(r) \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

5.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B < 2 \\ (1,1), & \text{if } \delta_A + \delta_B = 2 \end{cases}$$

This is supported when:

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

6.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (0,1), & \text{if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}$$

This is supported when:

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \max\{\phi(r), \gamma\}$$
$$\gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

7.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}$$

This is supported when:

$$\gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\phi(r), \gamma\}$$

8.

$$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0,\ \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}$$

This is supported when:

$$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

9.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \ \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$$

This is supported when:

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

10.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B = 0 \\ (1,1), \text{ if } \delta_A + \delta_B \geq 1 \end{cases}$$

This is supported when:

$$\phi(r) \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

11.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$

This is supported when:

$$\gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

Notice also that when $\lambda = (0,0)$, $d^\lambda(\delta) = (0,0)$, $\forall\delta$ is supported on all parameter values since the norm does not induce repayment.

Stage 1 best responses

The first stage best response depends on the chosen sub-game perfect repayment play. Here, the best responses under the 11 different repayment plays are explored.

1. If $d^\lambda(\delta) = (0,0)$, $\forall\delta$ is the repayment play:
   For A:

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big) - \gamma\delta_B$$

   Thus, $\delta_A^* = 0$ and by a similar argument, $\delta_B^* = 0$.
   This results in the following equilibrium profile that may arise in sub-games with any realization of λ

$$\big(\delta_A^* = \delta_B^* = 0; \ d^\lambda(\delta) = (0,0), \ \forall\delta\big)$$

   and the required parametric restrictions are:

- If $\lambda = (0,0)$, without further parametric restrictions
- If $\lambda \neq (0,0)$, then the parametric restrictions are

$$u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

2. If $d^\lambda(\delta) = (1,1),\ \forall\delta$ is the repayment play:
   For A:

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta < u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

   Thus, $\delta_A^* = 0$ and by a similar argument, $\delta_B^* = 0$.
   This results in sub-game perfect equilibrium profile

$$\big(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = (1,1),\ \forall\delta\big)$$

   and the required parametric restrictions are:
   $\lambda \neq (0,0)$, further parametric restrictions are

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

3. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$ is the repayment play:
   For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

   (when $\delta_B = 1$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta < u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

   Thus, $\delta_A^* = 0$ is dominant for A.
   For B (given $\delta_A^* = 0$):

$$U_B(\text{apply pressure}) \ \underline{?}\ U_B(\text{no pressure})$$
$$\text{depend on whether } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ \geq u\big(w + R\lambda_B\big)$$
$$\text{or } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ < u\big(w + R\lambda_B\big)$$

   There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\phi(r) - \eta, \gamma\}$$

4. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}$ is the repayment play:
A symmetric analysis to the above case yields that there are two equilibrium profiles in sub-games with $\lambda \neq (0,0)$:
   - $\left(\delta_A^* = 1, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}\right)$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

   - $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}\right)$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \max\{\phi(r) - \eta, \gamma\}$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

5. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B < 2 \\ (1,1), & \text{if } \delta_A + \delta_B = 2 \end{cases}$ is the repayment play:
For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ \underline{?}\ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ \underline{?}\ u\big(w + R\lambda_A\big) - \gamma$$

Thus, $\delta_A^*(\delta_B) = \delta_B$ if

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

and $\delta_A^*(\delta_B) = 0$ if

$$\phi(r) + \gamma - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

For B, the analysis is symmetric:
$\delta_B^*(\delta_A) = \delta_A$ if

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

and $\delta_B^*(\delta_A) = 0$ if

$$\phi(r) + \gamma - \eta < u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

There are thus five equilibrium profiles in sub-games with $\lambda \neq (0, 0)$ that may emerge:

- $\left(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B < 2 \\ (1,1), & \text{if } \delta_A + \delta_B = 2 \end{cases}\right)$ when

$$\begin{aligned} \phi(r) + \gamma &\geq u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) - u\big(w + R\lambda_A\big) > \gamma \\ \phi(r) + \gamma &\geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) > \gamma \end{aligned}$$

- $\left(\delta_A^* = \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B < 2 \\ (1,1), & \text{if } \delta_A + \delta_B = 2 \end{cases}\right)$ when

$$\begin{aligned} \phi(r) + \gamma - \eta &\geq u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) - u\big(w + R\lambda_A\big) > \gamma \\ \phi(r) + \gamma - \eta &\geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) > \gamma \end{aligned}$$

6. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (0,1), & \text{if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}$ is the repayment play:
   By symmetric analysis to the following case, there exist two equilibrium profiles in sub-games with $\lambda \neq (0, 0)$ that may emerge:

   - $\left(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right)$ when

$$\begin{aligned} \phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) &> \max\{\gamma, \phi(r) + \gamma - \eta\} \\ \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

   - $\left(\delta_A^* = \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right)$ when

$$\begin{aligned} \phi(r) + \gamma - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) &> \max\{\gamma, \phi(r)\} \\ \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

7. If $d^{\lambda}(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ > \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ > \ u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

Thus, $\delta_A^*(\delta_B) = \delta_B$.

For B (when $\delta_A = 0$):

$$U_B(\text{apply pressure}) < U_B(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_B\big) - \eta < u\big(w + R\lambda_B\big)$$

(when $\delta_A = 1$):

$$U_B(\text{apply pressure}) \ \underline{?}\ U_B(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ \underline{?}\ u\big(w + R\lambda_B\big) - \gamma$$

Thus, $\delta_B^*(\delta_A) = \delta_A$ when

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big)$$

and $\delta_B^*(\delta_A) = 0$ when

$$u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) \ > \phi(r) + \gamma - \eta$$

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = \delta_B^* = 0;\ d^{\lambda}(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right)$ when

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\gamma, \phi(r) + \gamma - \eta\}$$

- $\left(\delta_A^* = \delta_B^* = 1;\ d^{\lambda}(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right)$ when

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\gamma, \phi(r)\}$$

8. If $d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \ \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \ \underline{?} \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ \underline{?} \ u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ > \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ > \ u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

Thus, $\delta_A^*(\delta_B) = \delta_B$ when

$$\phi(r) - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

and $\delta_A^*(\delta_B) = 1$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

For B (when $\delta_A = 0$):

$$U_B(\text{apply pressure}) < U_B(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_B\big) - \eta < u\big(w + R\lambda_B\big)$$

(when $\delta_B = 1$):

$$U_B(\text{apply pressure}) \ < \ U_B(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ < \ u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r)$$

Thus, $\delta_B^*(\delta_A) = 0$

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 0; \ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}\right)$ when

$$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r) - \eta$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

- $\left(\delta_A^* = 1, \delta_B^* = 0; \ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}\right)$ when

$$\min\{\phi(r) - \eta, \ \gamma\} \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

9. If $d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \; \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \; \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$ is the repayment play:

   By symmetric analysis to the above case, the equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

   - $\left(\delta_A^* = 0, \delta_B^* = 0; \; d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \; \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \; \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases} \right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r) - \eta$$

   - $\left(\delta_A^* = 0, \delta_B^* = 1; \; d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \; \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \; \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases} \right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\min\{\phi(r) - \eta, \; \gamma\} \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

10. If $d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B = 0 \\ (1,1), \text{ if } \delta_A + \delta_B \geq 1 \end{cases}$ is the repayment play:

   For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \; \underline{?} \; U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \; \underline{?} \; u\big(w + R\lambda_A\big)$$

   (when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \; < \; U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \; < \; u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

   Thus, $\delta_A^*(\delta_B) = 1 - \delta_B$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

   and $\delta_A^*(\delta_B) = 0$ when

$$\phi(r) - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

   By symmetry, for B, $\delta_B^*(\delta_A) = 1 - \delta_A$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

   and $\delta_B^*(\delta_A) = 0$ when

$$\phi(r) - \eta < u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

   There are thus three equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 1, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}\right.$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

- $\left(\delta_A^* = 0, \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}\right.$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}\right.$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r) - \eta$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r) - \eta$$

11. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0, \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1, \delta_B = 0 \\ (1,0), & \text{if } \delta_A = 0, \delta_B = 1 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}$ is the chosen repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \leq U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \eta \leq u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ > \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ > \ u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

if and only if $\phi(r) > \eta$.
Thus, $\delta_A^*(\delta_B) = \delta_B$.
By symmetry, $\delta_B^*(\delta_A) = \delta_A$.
There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0, \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1, \delta_B = 0 \\ (1,0), & \text{if } \delta_A = 0, \delta_B = 1 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}\right)$ when

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

- $\left(\delta_A^* = 1, \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0, \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1, \delta_B = 0 \\ (1,0), & \text{if } \delta_A = 0, \delta_B = 1 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}\right)$ when

$$\begin{aligned} \gamma &\geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r) \\ \gamma &\geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r) \\ \phi(r) &\geq \eta \end{aligned}$$

Remark: households are not restricted to commit to a repayment play before the state has been realized.

On equilibrium path action profiles:

The analysis is conducted for sub-games starting at investment outcome realization λ:

For ease of notation, define:

$$u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \equiv X_A(\lambda)$$
$$u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \equiv X_B(\lambda)$$

1. $\delta_A = \delta_B = 0;\ d_A = d_B = 0$
   For all parametric values when $\lambda = (0,0)$
   When $\lambda \neq (0,0)$:
   - $X_A(\lambda) > \gamma$ and $X_B(\lambda) > \gamma$
   - $\phi(r) + \gamma \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda) > \max\{\gamma, \phi(r) - \eta\}$
   - $\phi(r) \geq X_A(\lambda) > \max\{\gamma, \phi(r) - \eta\}$ and $\phi(r) + \gamma \geq X_B(\lambda)$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma \geq X_B(\lambda) > \gamma$
   - $\gamma \geq X_A(\lambda)$ and $\phi(r) + \gamma \geq X_B(\lambda) > \max\{\gamma, \phi(r) + \gamma - \eta\}$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \max\{\gamma, \phi(r) + \gamma - \eta\}$ and $\gamma \geq X_B(\lambda)$
   - $\phi(r) \geq X_A(\lambda) > \phi(r) - \eta$ and $\phi(r) \geq X_B(\lambda) > \phi(r) - \eta$
   - $\gamma \geq X_A(\lambda) > \phi(r)$ and $\gamma \geq X_B(\lambda) > \phi(r)$
   - $\min\{\phi(r), \gamma\} \geq X_A(\lambda) > \phi(r) - \eta$ and $\phi(r) + \gamma \geq X_B(\lambda) > \phi(r)$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \phi(r)$ and $\min\{\phi(r), \gamma\} \geq X_B(\lambda) > \phi(r) - \eta$
2. $\delta_A = \delta_B = 0;\ d_A = d_B = 1$
   When $\lambda \neq (0,0)$:
   - $\phi(r) \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda)$
3. $\delta_A = 1, \delta_B = 0;\ d_A = d_B = 1$ When $\lambda \neq (0,0)$:
   - $\phi(r) - \eta \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma \geq X_B(\lambda)$
   - $\min\{\phi(r) - \eta,\ \gamma\} \geq X_A(\lambda)$ and $\phi(r) + \gamma \geq X_B(\lambda) > \phi(r)$
   - $\phi(r) - \eta \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda)$
4. $\delta_A = 0, \delta_B = 1;\ d_A = d_B = 1$ When $\lambda \neq (0,0)$:
   - $\phi(r) + \gamma \geq X_A(\lambda)$ and $\phi(r) - \eta \geq X_B(\lambda) > \gamma$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \phi(r)$ and $\min\{\phi(r) - \eta,\ \gamma\} \geq X_B(\lambda)$
   - $\phi(r) \geq X_A(\lambda)$ and $\phi(r) - \eta \geq X_B(\lambda)$
5. $\delta_A = \delta_B = 1;\ d_A = d_B = 1$ When $\lambda \neq (0,0)$:

- $\phi(r) + \gamma - \eta \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma - \eta \geq X_B(\lambda) > \gamma$
- $\gamma \geq X_A(\lambda)$ and $\phi(r) + \gamma - \eta \geq X_B(\lambda) > \max\{\phi(r), \gamma\}$
- $\phi(r) + \gamma - \eta \geq X_A(\lambda) > \max\{\phi(r), \gamma\}$ and $\gamma \geq X_B(\lambda)$
- $\gamma \geq X_A(\lambda) > \phi(r)$ and $\gamma \geq X_B(\lambda) > \phi(r)$ and $\phi(r) \geq \eta$ ▲

Setting $\phi(\cdot, \cdot, \cdot)$:

The continuation values for different lottery choices are now established.

Both play safe

$$\phi(r, L_{safe}, L_{safe})_1 = \beta\Big\{\mu\big[u(w + R - r)\big] + (1-\mu)\big[u(w + 0.5 \cdot R - r)\big] + \phi(r, L_{safe}, L_{safe})_1\Big\}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big]$$

$$\phi(r, L_{safe}, L_{safe}) = \phi(r, L_{safe}, L_{safe})_1 - \phi_0$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta}$$

$$\phi(r, L_{safe}, L_{safe})_1 = \frac{\beta EU^{SS}(norm)}{1-\beta}$$

where

$$EU^{SS}(norm) = \mu\big[u(w + R - r)\big] + (1-\mu)\big[u(w + 0.5 \cdot R - r)\big]$$

and hence

$$\phi(r, L_{safe}, L_{safe}) = \frac{\beta\big[EU^{SS}(norm) - u(w)\big]}{1-\beta}$$

Both go risky

$$\begin{aligned}\phi(r, L_{risky}, L_{risky})_1 = &\beta\Big\{\mu^2\big[u(w + 2R - r) + \phi(r, L_{risky}, L_{risky})_1\big] \\ &+ \mu(1-\mu)\big[u(w + 2R - (2 - 0.5\theta)r) + \phi(r, L_{risky}, L_{risky})_1\big] \\ &+ (1-\mu)\mu\big[u(w - 0.5\theta \cdot r) + \phi(r, L_{risky}, L_{risky})_1\big] + (1-\mu)^2\big[u(w) + \phi_0\big]\Big\}\end{aligned}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big]$$

$$\phi(r, L_{risky}, L_{risky}) = \phi(r, L_{risky}, L_{risky})_1 - \phi_0$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta}$$

$$\phi(r, L_{risky}, L_{risky})_1 = \frac{\beta EU^{RR}(norm) + \beta\phi_0(1-\mu)^2}{1-\beta\mu(2-\mu)}$$

where

$$EU^{RR}(norm) = \mu^2\big[u(w+2R-r)\big] + \mu(1-\mu)\big[u(w+2R-(2-0.5\theta)r)\big] \\ + (1-\mu)\mu\big[u(w-0.5\theta r)\big] + (1-\mu)^2\big[u(w)\big]$$

and hence

$$\phi(r, L_{risky}, L_{risky}) = \frac{\beta EU^{RR}(norm) + (\beta-1)\phi_0}{1-\beta\mu(2-\mu)}$$

A goes risky - B goes safe

$$\phi(r, L_{risky}, L_{safe})_1 = \beta\Big\{\mu^2\big[u(w+2R-r) + \phi(r, L_{risky}, L_{safe}))_1\big] \\ + \mu(1-\mu)\big[u(w+2R-r) + \phi(r, L_{risky}, L_{safe}))_1\big] \\ + (1-\mu)\mu\big[u(w-\theta r) + \phi(r, L_{risky}, L_{safe}))_1\big] \\ + (1-\mu)^2\big[u(w-2\theta r) + \phi(r, L_{risky}, L_{safe}))_1\big]\Big\}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big]$$

$$\phi(r, L_{risky}, L_{safe})) = \phi_1^{RS} - \phi_0$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta}$$

$$\phi(r, L_{risky}, L_{safe}))_1 = \frac{\beta EU^{RS}(norm)}{1-\beta}$$

where

$$EU^{RS}(norm) = \mu\big[u(w+2R-r)\big] + (1-\mu)\mu\big[u(w-\theta r)\big] + (1-\mu)^2\big[u(w-2\theta r)\big]$$

and hence

$$\phi(r, L_{risky}, L_{safe})) = \frac{\beta\big[EU^{RS}(norm) - u(w)\big]}{1-\beta}$$

A goes safe - B goes risky

$$\begin{aligned}\phi(r, L_{safe}, L_{risky})_1 = &\beta\Big\{\mu^2\big[u(w+R-r)+\phi(r, L_{safe}, L_{risky})_1\big]\\ &+\mu(1-\mu)\big[u(w+R-(2-\theta)r)+\phi(r, L_{safe}, L_{risky})_1\big]\\ &+(1-\mu)\mu\big[u(w+0.5R-r)+\phi(r, L_{safe}, L_{risky})_1\big]\\ &+(1-\mu)^2\big[u(w+0.5R-(2-\theta)r)+\phi(r, L_{safe}, L_{risky})_1\big]\Big\}\end{aligned}$$

$$\phi_0 = \beta\Big[u(w)+\phi_0\Big]$$

$$\phi(r, L_{safe}, L_{risky}) = \phi_1^{SR} - \phi_0$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta}$$

$$\phi(r, L_{safe}, L_{risky})_1 = \frac{\beta EU^{SR}(norm)}{1-\beta}$$

where

$$\begin{aligned}EU^{SR}(norm) = &\mu^2\big[u(w+R-r)\big]+\mu(1-\mu)\big[u(w+R-(2-\theta)r)\big]\\ &+(1-\mu)\mu\big[u(w+0.5R-r)\big]+(1-\mu)^2\big[u(w+0.5R-(2-2\theta)r)\big]\end{aligned}$$

and hence

$$\phi(r, L_{safe}, L_{risky}) = \frac{\beta\big[EU^{SR}(norm)-u(w)\big]}{1-\beta}$$

▲