**ECON 6100** 5/7/2021

# Section 11

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Adopted From Fikri's material

## 1 Review - NTU

$(v_w, v_f)$

In *Non-Transferable Utility (NTU) Matching* problem, the set up includes a set of men $M$ and a set of women $W$ with each one having a strict ranking over the other group. Without loss of generality we can assume $|M| = |W|$.

**Example 1.** Let $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$, then $m_1$ has a ranking over $w_1$ and $w_2$, say, $w_1 \succ_{m_1} w_2$. $m_2$ also has ranking $w_1 \succ_{m_2} w_2$ and so do each women, $m_1 \succ_{w_1} m_2$ and $m_1 \succ_{w_2} m_2$.

The goal is to find a match, i.e. $m \leftrightarrow w$ so that everyone is "happy" in the sense that no man and woman would want to switch partners. More formally, a match is stable if there is no $m$ and $w'$ such that: $m \leftrightarrow w$, $m' \leftrightarrow w'$ and $w' \succ_m w$ and $m \succ_{w'} m'$. At first sight it is unclear that a stable match exist for any arbitrary preferences, but then...

> One of Mr Shapley's better-known achievements is the Gale-Shapley matching algorithm, which he devised after an old university friend (David Gale) asked for help to solve a problem. Given two groups of people, each with slightly different preferences, is there a way to match them in such a way that people aren't constantly ditching their partner? After much head-scratching, Mr Gale suspected there was no solution, but could not prove it. As Mr Shapley told it, the solution took him the best part of an afternoon.—*The Economist*[1]

The Gale-Shapley Algorithm, also known as the Deferred Acceptance Algorithm (DAA) is as follows:

1. Each man proposes to his top-ranked choice
2. Each woman keeps her top-ranked man among those that proposed to her and rejects the rest.
3. Each man who has been rejected proposes to his top-ranked choice among those who have not rejected him.
4. Each woman keeps her best proposal among the new ones and the one from previous round and rejects the rest.
5. The process ends no man has a woman to propose to.

This is a men proposing version of the algorithm. There is an analogous women proposing algorithm. It is not hard to see that the DAA ends in a stable match. It is also true, but harder to see, that the stable match given from the men proposing DAA is male-optimal in the sense that there is no other stable matching such that any $m$ gets matched with a higher-ranked woman.

**Example 2.** In our example, in the first round $m_1$ and $m_2$ both propose to $w_1$. $w_1$ rejects $m_2$. In round 2, $m_2$ proposes to $w_2$. The algorithm ends with $m_1 \leftrightarrow w_1$ and $m_2 \leftrightarrow w_2$. This is stable since even though $m_2$ prefers to be matched with $w_1$, $w_1$ is happy with her current match $m_1$. Similarly for $w_2$.

[1] http://www.economist.com/blogs/freeexchange/2016/03/matchmaker-heaven

## 2 Problems

**Problem 1.** In this matching problem we assume that getting married is strictly preferred by all men and women to staying single. Consider the following preferences for 5 men and 5 women:

| | | | |
|---|---|---|---|
| $m_1$ | 2, 3, 4, 5, 1 | $w_1$ | 1, 2, 3, 5, 4 |
| $m_2$ | 3, 4, 5, 1, 2 | $w_2$ | 2, 1, 4, 5, 3 |
| $m_3$ | 5, 1, 4, 2, 3 | $w_3$ | 3, 2, 5, 1, 4 |
| $m_4$ | 3, 1, 2, 4, 5 | $w_4$ | 4, 5, 1, 2, 3 |
| $m_5$ | 1, 5, 2, 3, 4 | $w_5$ | 5, 1, 2, 3, 4 |

(a) Using the men-proposing DAA, find a stable match.

(b) Using the women-proposing DAA, find a stable match.

(c) When the men-proposing version is used, can women 1 be better off by not revealing her true preferences?

Strategy Proofness

a) Male proposing DAA

R1:

$m_1 \rightarrow w_2$ ✓
$m_2 \rightarrow w_3$ ✓
$m_3 \rightarrow w_5$ ✓
$m_4 \rightarrow w_3$ ✗
$m_5 \rightarrow w_1$ ✓

R2:

$m_4 \rightarrow w_1$ ✗
$m_5 \rightarrow w_1$ ✓

R3:

$m_4 \rightarrow w_2$ ✗
$m_1 \rightarrow w_2$ ✓
$m_3 \rightarrow w_5$ ✓
$m_5 \rightarrow w_1$ ✓
$m_2 \rightarrow w_3$ ✓

R4:

$m_4 \rightarrow w_4$ ✓
$m_1 \rightarrow w_2$ ✓
$m_3 \rightarrow w_5$ ✓
$m_5 \rightarrow w_1$ ✓
$m_2 \rightarrow w_3$ ✓

(b) R1:

$w_1 \to m_1$ ✓, $w_2 \to m_2$ ✓, $w_3 \to m_3$ ✓, $w_4 \to m_4$ ✓, $w_5 \to m_5$ ✓

(c) Let's suppose: $w_1 : 1 \succ 2 \succ 3 \succ 4 \succ 5$

R2* (new round 2):

$m_4 \to w_1$
$m_1 \to w_2$
$m_2 \to w_3$
$m_3 \to w_5$

R3*:

$m_5 \to w_5$ ✓
$m_3 \to w_3$ ✗.

R4*

$m_1 \quad w_4$
$m_2 \quad w_3$
$m_3 \to w_1$ ✓
$m_4 \to w_1$ ✗
$m_5 \to w_5$

R5*:

$m_1 \quad w_4$
$m_2 \quad w_3$
$\boxed{m_3 \quad w_1}$
$m_4 \quad w_2$
$m_5 \quad w_5$.

## 3 Arrow-Debreu Equilibrium

The set up of the exchange economy we have seen is the following. There are $L$ goods and $N$ individuals. Consumption of individual $i$ is denoted by $x^i \in \mathbb{R}^L_+$. Endowment is denoted by $\omega^i \in \mathbb{R}^L_+$. Each individual has a preference $\succsim_i$ over the goods represented by $u^i : \mathbb{R}^L_+ \to \mathbb{R}$. Equilibrium happens when all individuals solve their UMP's and markets clear.

We now introduce uncertainty into the model. This can be done by having $S$ states of nature, only one of which will occur. We start with the Arrow-Debreu model. The key to Arrow-Debreu is that we can think of good $l$ in state $s$ as a different good from good $l$ in state $s'$. This is the notion of *state-contingent commodities*. All decisions in the model happen before the state of the world is realized, so formally, the model can be thought of as having two time periods $t = 0$, where individuals solve their UMP's, and $t = 1$, where the state is realized and trade happens.

With this set up, individuals are not thinking about how much of the $L$ goods to consume, but rather how much of the $L + LS = L(S+1)$ contingent commodities to consume[2]. The consumption and endowment vectors for individual $i$ are denoted by $x^i \in \mathbb{R}^{L(S+1)}_+$ and $\omega^i \in \mathbb{R}^{L(S+1)}_+$. Prices are denoted by $p \in \mathbb{R}^{L(S+1)}_{++}$.

What about utility? Individuals still have utility function $u^i$ on the $L$ physical goods as before, but they have subjective beliefs on which state will occur $\pi^i$ on $S$. However, they may deal with uncertainty in period 1 in different ways. That is, each $i$ takes $u^i$ and $\pi^i$, and marry them in some way. For example if $i$ is an expected utility maximizer, then $i$'s utility function over the contingent commodities is: $V_i = u^i(x^i_{10}, \ldots, x^i_{L0}) + \sum_s \pi^i_s u^i(x^i_{1s}, \ldots, x^i_{Ls})$. The individual may be pessimistic and only value future consumption in worst state: $\min_s u^i(x^i_{1s}, \ldots, x^i_{Ls})$. We can also have the case where $i$ has multiple $\pi^i$'s in $\Pi^i$ and $i$ is a maximin expected utility maximizer over period 1 contingent commodities (Gilboa-Schmeidler): $\min_{\pi^i \in \Pi^i} \sum_s \pi^i_s u^i(x^i_{1s}, \ldots, x^i_{Ls})$.

To summarize, the ingredients of the Arrow-Debreu model are:

- $L$ physical goods, $S$ states of the world, $N$ individuals.
- $x^i \in \mathbb{R}^{L(S+1)}_+$, $\omega^i \in \mathbb{R}^{L(S+1)}_+$, $p \in \mathbb{R}^{L(S+1)}_{++}$.
- $u^i : \mathbb{R}^L_+ \to \mathbb{R}$ and $\Pi^i = \{\pi^i = (\pi^i_1, \ldots, \pi^i_S) \gg 0, \sum_s \pi^i_s = 1\}$, and some way of combining them.

The bottom line is that with all this formalism, the Arrow-Debreu economy is just an exchange economy with more goods and more elaborate utility functions $V_i$'s over the contingent commodities. Equilibrium is the price system and allocation when individuals solve their UMP's:

$$\max_x V^i(x) \quad \text{subject to} \quad x \in \mathcal{B}^i_{AD}(p, \omega^i) = \left\{x \in \mathbb{R}^{L(S+1)}_+ : p \cdot x \leq p \cdot \omega^i\right\}$$

and the market for all contingent commodities clear, $\sum_i x^i = \sum_i \omega^i$. Let's denote the **Arrow-Debreu equilibrium** by $x^* = (x^{i*})_{i=1}^N \in \mathbb{R}^{L(S+1)N}$ and $p^* \in \mathbb{R}^{L(S+1)}_{++}$. In the Arrow-Debreu model, all trade takes place in period 0, before the state is realized, so the equilibrium is determined in period 0.

[2]Some books, for instance MWG, do not allow individuals to consume in period 0, though this is usually not explicitly mentioned. This is a special case of our set up with everyone having zero endowment in period 0.

## 4 Radner Equilibrium

The idea of a Radner economy, or a financial economy as it is often called, is that in addition to physical goods and states, you also have tradable *financial assets*. Suppose there are $J$ financial assets with asset $j$ paying out $a_{sj} \geq 0$ in state $s$. We can write the $S \times J$ asset return matrix $A$ as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1J} \\ \vdots & a_{22} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{S1} & \dots & \dots & a_{SJ} \end{bmatrix}$$

Let $z^i \in \mathbb{R}^J$ denotes the portfolio of individual $i$, i.e., $i$ holds $z^i_j$ units of asset $j$ (note that this may be negative), then the payout in state $s$ is $(Az^i)_s$, the $s$-th row of $Az^i$. We say that the market is complete if $A$ has full row rank, i.e., $\text{rank}(A) = S$. For example, for two assets and two states we might have

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Let $z^i = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, then $Az^i = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$. If state 1 occurs, $i$ gets return 5, while if state 2 occurs, $i$ gets return 0. The market is complete in this case.

We denote the prices of assets by $r$ and to distinguish from the Arrow-Debreu prices, denote the prices of physical state-contingent commodities by $q$. To summarize, the ingredients of the Radner (financial economy) model are:

- $L$ physical goods, $S$ states of the world, $N$ individuals.
- $x^i \in \mathbb{R}^{L(S+1)}_+$, $\omega^i \in \mathbb{R}^{L(S+1)}_+$, $q \in \mathbb{R}^{L(S+1)}_{++}$.
- $z^i \in \mathbb{R}^J$, $A_{S\times J}$ with $a_{sj} \geq 0$, and $r \in \mathbb{R}^J_{++}$
- $u^i : \mathbb{R}^L_+ \to \mathbb{R}$ and $\Pi^i = \{\pi^i = (\pi^i_1, \dots, \pi^i_S) \gg 0, \sum_s \pi^i_s = 1\}$, and some way of combining them.

In addition to assets, the timing of trade is the key feature of the Radner model. There are still two time periods. In period 0, physical commodities and assets are traded. The budget constraint in period 0 is

$$q_0 \cdot x^i_0 + r \cdot z^i \leq q_0 \cdot \omega^i_0$$

In period 1, which depends on the realized state, you can trade physical commodities. Suppose state $s$ is realized, individual $i$'s income in period 1 comes from selling $i$'s endowment in state $s$ plus the return from portfolio $z^i$. The budget constraint in state $s$ is

$$q_s \cdot x^i_s \leq q_s \cdot \omega^i_s + (Az^i)_s$$

As we have defined, the return of the asset is in monetary unit, for this to make sense in each market we need to have a numeraire good. The convention is to take the first good in each state as the numeraire. In other words, we take $q_{s1} = 1$.

The UMP of individual $i$ is

$$\max_{x,z} V^i(x) \quad \text{subject to} \quad (x,z) \in \mathcal{B}^i_R(q,r,\omega^i)$$

$$\mathcal{B}^i_R(q,r,\omega^i) = \left\{(x,z) \in \mathbb{R}^{L(S+1)}_+ \times \mathbb{R}^J : q_0 \cdot x^i_0 + r \cdot z^i \leq q_0 \cdot \omega^i_0 \text{ and } q_s \cdot x^i_s \leq q_s \cdot \omega^i_s + (Az^i)_s \text{ for all } s \in S\right\}$$

A **Radner equilibrium** is $(x^*, z^*, q^*, r^*)$ such that

1. $(x^{i*}, z^{i*}) \in \arg\max_{(x,z)\in\mathcal{B}^i_R(q,r,\omega^i)} V^i(x)$, everyone solves UMP.
2. $\sum_i x^{i*} = \sum_i \omega^i$, goods market clear.
3. $\sum_i z^{i*} = 0$, financial market clears.

# Notes

This material is in MWG Chapter 19 A-E. Another good resource online is Prof. Alberto Bisin's note on General Equilibrium Theory at `https://www.econ.nyu.edu/user/bisina/Lecture%20notes%20Oct2014.pdf`.

# Example for Arrow - Debreu:

1 good : food (F)

2 consumers : A and B

At T=1 — 2 states : sunny (S) and rainy (R)

$u^A_{T,s}$ or $u^B_{T,s} : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a felicity function that represent consumption preferences.

Key features of Arrow-Debreu model:

$\boxed{1}$ Consumers choose $x^i(T=0)$, $x^i(T=1,S)$, $x^i(T=1,R) \in \mathbb{R}_+$

In summary, $x^i \in \mathbb{R}_+^{(1+2)}$ $\boxed{\text{chosen at } T=0}$.

$\boxed{2}$ Consumers' endowments: $\omega^i(T=0)$, $\omega^i(T=1,S)$, $\omega^i(T=1,R) \in \mathbb{R}^2_+$

In summary, $\omega^i \in \mathbb{R}_+^{(1+2)}$

$\boxed{3}$ Three sets of ~~vaccines~~ prices after $p(T=0)$, $p(T=1,S)$, $p(T=1,R) \in \mathbb{R}_{++}$.

So, $p \in \mathbb{R}_{++}^{(1+2)}$

$\boxed{4}$ There exists some known probability distribution over states

$$\pi^i_S + \pi^i_R = 1 \quad , \forall i \in \{A, B\}$$

Agents are expected utility maximizers: (Alternates possible)

$$V_i(x^i) = u^i_0(x^i(T=0)) + \pi^i_S u^i_S(x^i(T=1,S)) + \pi^i_R u^i_R(x^i(T=1,R))$$

An equilibrium corresponds to

i - Utility maximization, $\forall i$

UMP corresponds to:

$$\max_{x \in \mathbb{R}_+^{2(1+2)}} V_i(x) \quad \text{s.t.} \quad \overbrace{\underbrace{\underset{3\times1}{p} \cdot \underset{3\times1}{x} \leq \underset{3\times1}{p} \cdot \underset{3\times1}{\omega^i}}}$$

$\because$ x is chosen at T=0.
"Contingent claims"

ii - Market clearing:

$$\underset{3\times1}{x^A} + \underset{3\times1}{x^B} = \underset{3\times1}{\omega^A} + \underset{3\times1}{\omega^B}$$

(3 constraints)

F in (T=0), (T=1, S) & (T=1, R).

Suppose $u^A_{T=1}(y) = u^B_{T=1}(y) = \ln(y)$ and $\pi_S^A = \pi_R^A = 1/2$. ✓

⊛ No T=0 felicity! $u^i_{T=0}(y) = 0$

$\pi_S^B = 1/3$, $\pi_R^B = 2/3$

$\omega^A = (0, 2, 1)$, $\omega^B = (0, 1, 2)$. ✓

To find the eqbm:

i - UMP(A): $\max \frac{1}{2} \ln(x_S^A) + \frac{1}{2} \ln(x_R^A)$

$$\text{s.t. } \underbrace{p_0 x_0^A}_{0} + p_S x_S^A + p_R x_R^A \underset{=}{\leq} 2p_S + p_R$$

$$\therefore\ x_S^A = \frac{2p_S + p_R}{2p_S}\ ;\ x_R^A = \frac{2p_S + p_R}{2p_R}$$

Similarly UMP(B) yields:

$$x_S^B = \frac{p_S + 2p_R}{3p_S}\ ;\ x_R^B = \frac{2(p_S + 2p_R)}{3p_R}. ✓$$

ii -

$$\left.\begin{aligned} x_S^A + x_S^B &= 3 \\ x_R^A + x_R^B &= 3 \end{aligned}\right\}$$

Plug Marshallian dem

An eqbm is $(p_0, p_S, p_R) = (1, 1, 10/7)$, $x^A = (0, 12/7, 12/10)$

$x^B = (0, 9/4, 18/10)$.

1 ✓ ?

# Radner equilibrium:

In addition to Food, consumers can buy securities / assets

Suppose there are 2 assets:

i- Bond :- returns 1F in each state.

ii- Insurance - returns 1F only in rain.

Represent this in a return matrix

$$A = \begin{array}{c c} & \begin{array}{cc} \text{Bond} & \text{Ins} \end{array} \\ \begin{array}{c} S \\ R \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array} \longrightarrow \text{Linearly independent!}$$

In this model: (can only trade goods in states they are realized)

$x^i \in \mathbb{R}^3_+ \rightarrow$ allocation of food in $(T=0)$, $(T=1, S)$, $(T=1, R)$

$\omega^i \in \mathbb{R}^3_+ \rightarrow$ endowments

$q \in \mathbb{R}^3_{++} \rightarrow$ prices of physical assets * Normalize $(q_0)_1 = 1$

$(q_S)_1 = (q_R)_1 = 1$.

Short. $\leftarrow z^i \in \mathbb{R}^2 \rightarrow$ asset position / holding

$\rightarrow r \in \mathbb{R}^2_{++} \rightarrow$ asset prices

Equilibrium is composed of:

i- UMP: $\max\limits_{x, z} V_i(x)$ ✓

$$\text{s.t.} \quad q_0 x_0 + r \cdot z^i \leq q_0 \omega_0^i \quad T=0$$

$$q_S x_S \leq \boxed{q_S \omega_S^i} + (A \cdot z^i)_S \quad T=1, S$$

$$q_R x_R \leq q_R \omega_R^i + (A \cdot z^i)_R.$$

ii - Market clearing

$x^A + x^B = \omega^A + \omega^B$ ✓

$z^A + z^B = 0$ ✓

Let's plug this back in our earlier example:

i- UMP(A): $\max_{x^A, z^A} \frac{1}{2}\ln(x_S^A) + \frac{1}{2}\ln(x_R^A)$ ✓ $z_B^A = -\frac{q_I}{q_B} z_{Ins}^A$

s.t. $q_{Bond}\, z_{Bond}^A + q_{Ins} \cdot z_{Ins}^A \leq 0 \rightarrow$

$x_S^A \leq 2 + z_{Bond}^A$

$x_R^A \leq 1 + z_{Ins}^A + z_{Bond}^A$

$\Leftrightarrow \max_{z_{Bond}^i} \frac{1}{2}\ln(2 + z_{Bond}^A) + \frac{1}{2}\ln\left(1 + z_{Bond}^A(1 - q_1/q_2)\right)$.

ii- $x_S^A + x_S^B = 3$, $x_R^A + x_R^B = 3$

$z_{Bond}^A = -z_{Bond}^B$, $z_{Ins}^A = -z_{Ins}^B$

So, the solution here is:

$z_{Bond}^A = -4/14$, $z_{Bond}^B = 4/14$ $\}$ $+ = 0$

$z_{Ins}^A = 17/35$, $z_{Ins}^B = -17/35$ $\}$ $+ = 0$

$x^A = (12/7, 12/10)$

$x^B = (9/7, 18/10)$ $\} \longrightarrow$ Identical to A.D.

## 5 Problems

**Problem 2.** Suppose consumers are trading contingent claims on a single commodity, say sheep. States are in $S = \{a, b\}$. The aggregate endowment of sheep is independent of the state, but each individual's endowment of sheep is state-dependent. Suppose that individual 1 has preference of Gilboa-Schmeidler form. Individual 1 has a closed set of probability distributions $P$, and for any contract $c \in \mathbb{R}_+^{|S|}$,

$$V(c) = \min_{p \in P} \sum_{s \in S} u(c(s))p(s)$$

where $u$ is a concave payoff function. Individual 2 is a risk-averse expected utility maximizer with belief distribution $q$.

(a) Let $P$ contain two probability distributions $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{3}{4}, \frac{1}{4})$, and let $q = (\frac{1}{2}, \frac{1}{2})$. Draw an indifference curve for individual 1 corresponding to a particular utility level $\bar{u}$. Do the same for individual 2.

(b) Describe the set of Pareto optimal allocations.

(c) If $(p, x_1, x_2)$ is a competitive equilibrium, what can you say about $p$?

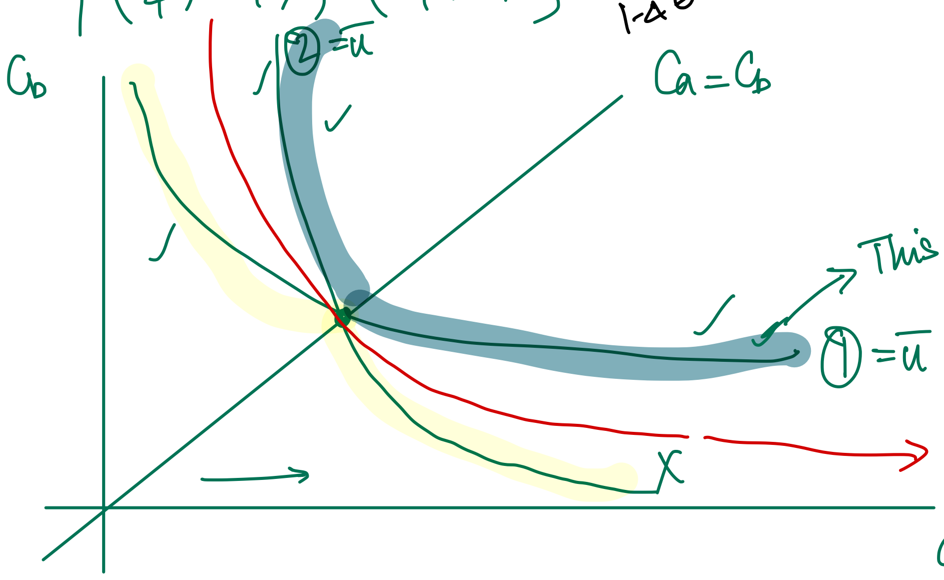
$V(c) = \min \left\{ \frac{u(c_A) + u(c_B)3}{4}, \frac{3}{4}u(c_A) + \frac{u(c_B)}{4} \right\}$ ①, ②.

$p_a, p_b$

$p_a = p_b$

(a) $P = \{(1/4, 3/4), (3/4, 1/4)\}$

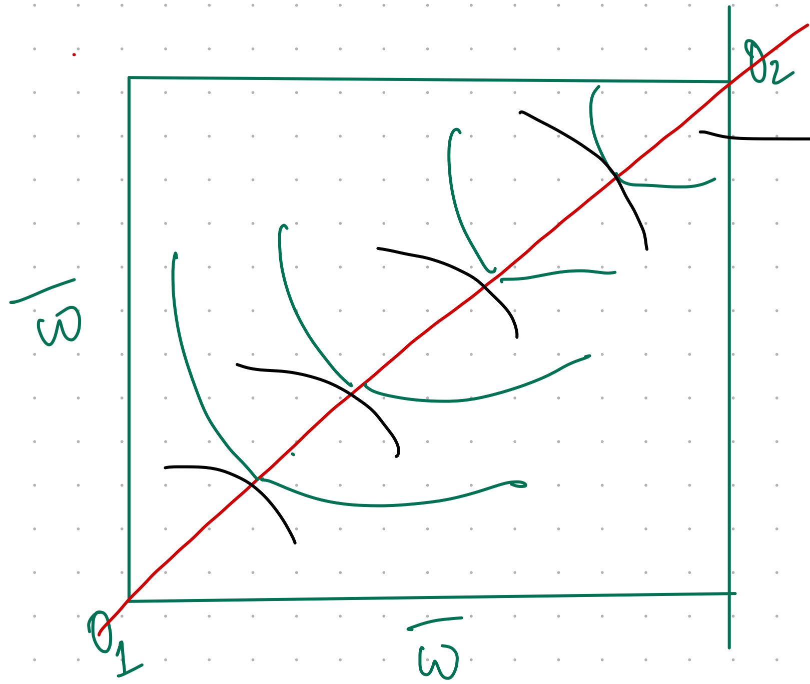
$\Delta \in P$
$1-\Delta \in P$




This = $V(c) = \bar{u}$

$V^2(c) = \bar{u}$ IC for user 2.

$c_a > c_a$.




$\Rightarrow$ Diagonal is the only place IC's intersect

$\Rightarrow$ PO allocations.

$C_A = C_B$

**Problem 3.** Consider an exchange economy with two consumers. In period 0 consumers consume nothing and trade only financial assets. In period 1 there are two possible states, $H$ and $T$. A single good is available in each state. The aggregate endowment is 3 in each state. Consumer $A$ is endowed with 2 units in state $H$ and 1 unit in state $T$. Consumer $B$ is endowed with the rest. Both consumers believe each state is equally likely. Utility over second period consumption for $A$ and $B$ are

$$u^A(x_H, x_T) = \frac{1}{2}v^A(x_H) + \frac{1}{2}v^A(x_T)$$
$$u^B(x_H, x_T) = \frac{1}{2}v^B(x_H) + \frac{1}{2}v^B(x_T)$$

The function $v^A$ and $v^B$ are strictly concave, increasing, differentiable and satisfy Inada conditions. There are two assets available for trading in period 0. The first asset pays 1 unit in each state, The second asset pays 1 unit in $H$ and $a \neq 1$ unit of good in state $T$.

(a) For all $a \neq 1$ and $a > 0$, compute the Radner equilibrium.

(b) What happens to equilibrium if $a = 1$?

$$A = \begin{array}{c c} & \begin{array}{cc} A1 & A2 \end{array} \\ \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix} \end{array} \quad a \neq 1.$$

$$(A \cdot Z^A)_H = 1Z_1^A + 1Z_2^A$$

(a) UM + Market clearing

UMP-A: $\max_{x^A, Z^A} \frac{1}{2} v^A(x_H^A) + \frac{1}{2} v^A(x_T^A)$

s.t. $q_1 \cdot Z_1^A + q_2 \cdot Z_2^A \leq 0 \quad (T=0)$

$x_H^A \leq 2 + Z_1^A + Z_2^A \quad (T=1, H)$

$x_T^A \leq 1 + Z_1^A + aZ_2^A \quad (T=1, T)$

$$Z_2^A = -\frac{q_1}{q_2} Z_1^A$$

$$\Longleftrightarrow \max_{Z_1^A} \frac{1}{2} v^A\underbrace{\left(2 + \left(1 - \frac{q_1}{q_2}\right) Z_1^A\right)}_{Y_H} + \frac{1}{2} v^A \underbrace{\left(1 + \left(1 - \frac{aq_1}{q_2}\right) Z_1^A\right)}_{Y_T}$$

FOC: $\left(1-\frac{q_1}{q_2}\right) v^{\prime A}(Y_H) = -\left(1-\frac{a q_1}{q_2}\right) v^{\prime A}(Y_T)$.

UMP(B):

FOC: $\left(1-\frac{q_1}{q_2}\right) v^{\prime B}\left(1+\left(1-\frac{q_1}{q_2}\right) z_1^B\right)$

$$= -\left(1-\frac{a q_1}{q_2}\right) v^{\prime B}\left(2+\left(1-\frac{a q_1}{q_2}\right) z_1^B\right).$$

In eqbm:

$$-\frac{(1-q_1/q_2)}{1-aq_1/q_2} = \frac{v_H^{\prime A}}{v_T^{\prime A}} = \frac{v_H^{\prime B}}{v_T^{\prime B}}$$

$v^A, v^B$ str conc. + $\overline{w}$ are equal.

$$\boxed{-\left(1-\frac{q_1}{q_2}\right) = 1-\frac{a q_1}{q_2}}$$

Towards a contr. if $\boxed{>}$

$$v_H^{\prime A} > v_T^{\prime A}.$$

$$x_H^A < x_T^A$$

$$+ \qquad +$$

$$x_H^B < x_T^B.$$

$$=3 \qquad =3 \qquad \text{(By MC)}$$

$$\boxed{\frac{q_1}{q_2} = \frac{2}{1+a}}$$ FA prices

$$z_1^A = \frac{1+a}{2(1-a)} \; ; \; z_2^A = \frac{1}{a-1}.$$

$$z_1^B = -z_1^A \; ; \; z_2^B = -z_2^A$$

F.A. alloc.

$$\boxed{x^A = \left(\tfrac{3}{2}, \tfrac{3}{2}\right) \; ; \; x^B = \left(\tfrac{3}{2}, \tfrac{3}{2}\right).}$$

P.A. alloc.

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$a = 1.$

(b) $q_1 = q_2$

$$z_1^A + z_2^A = 0 \qquad x^A = w^A$$

$$z_1^B + z_2^B = 0 \qquad x^B = w^B$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$\rightarrow$ Incomplete market.