ECON 6100	3/26/2021
Section 6	
Lecturer: Larry Blume	*TA: Abhi Ananth*

1 Review

An equilibrium can be though to be comprised of three components:

1. What gets consumed?
2. What gets produced?
   (a) **What goods are produced and how much?**
   (b) What factors are used and how much?
3. What prices make this exchange work?

1.1 Two sector models

The economy is endowed with two production processes (sectors) f_A and f_B that produce goods A and B respectively. It is a two sector model because each sector produces a unique good.

We typically assume there are two factors of production, capital (k) and labor (l) that move freely between the two sectors.

The production functions are assumed to satisfy:

A.1 The production function f_j is twice continuously differentiable with $f_j' > 0$ and $f_j'' < 0$.

A.2 The production function satisfies Inada condition (this is important because ... *who has the time to check for corner solutions?*)

$$\lim_{k \to 0} \frac{\partial f_j(k,l)}{\partial k} = \lim_{l \to 0} \frac{\partial f_j(k,l)}{\partial l} = +\infty \tag{1}$$

A.3 f_j is homogenous of degree 1 (constant returns to scale).

1.2 HOV model

Consider a small open economy trading goods A and B in a large world market. Consequently, prices p_A and p_B are determined independently of production here. The economy is endowed with endowments of factor inputs K and L.

1.2.1 Producer feasibility and efficiency

The PPS can be written as:

$$PPS = \{(y_A, y_B) : \; y_j \leq f_j(k_j, l_j) \; \forall j \in \{A, B\}, k_A + k_B \leq K, l_A + l_B \leq L\} \tag{2}$$

The production possibility frontier (PPF) are the set of all (y_A, y_B) pairs that simultaneously solves:

$$\begin{gathered}\phi(y_B) = \max f_A(k_A, l_A) \\ \text{s. t. } \; f_A(k_A, l_A) \leq y_B \\ k_A + k_B \leq K \\ l_A + l_B \leq L\end{gathered}$$

and

$$\begin{gathered}\phi(y_A) = \max f_B(k_B, l_B) \\ \text{s. t. } \; f_B(k_B, l_B) \leq y_A \\ k_A + k_B \leq K \\ l_A + l_B \leq L\end{gathered}$$

1.2.2 Equilibrium

Define any equilibrium as $\left(w^*, r^*, (l_j^*, k_j^*, y_j^*)_{j \in \{A,B\}}\right)$ such that:

1. $(l_j^*, k_j^*, y_j^*)_{j \in \{A,B\}}$ maximizes profit:

$$l_j^*, k_j^*, y_j^* = \arg\max p_j y_j - r k_j - w l_j \, \text{s. t. } \; y_j \leq f(k_i, l_i) \tag{3}$$

2. $\sum_j (l_j^*, k_j^*) = (L, K)$

We know from lecture that there are two classes of equilibria here:

- Diversified

- Specialized

But first, recall that Shepard's lemma gives us the factor use as the gradient of the unit cost functions:

$$l_j = \frac{\partial c_j(w,r)}{\partial w},\ k_j = \frac{\partial c_j(w,r)}{\partial r} \tag{4}$$

2 Problems

Problem 1. Suppose in a small open economy, world output prices for good 1 and 2 are p and 1, respectively. The production functions are:

$$q_1 = k^{1/2}l^{1/2}$$
$$q_2 = k^{3/4}l^{1/4}$$

(a) Compute the equilibrium factor prices.

(b) Compute $\frac{\partial w}{\partial p}$ when both goods are produced.

(c) Suppose $p = 1$. If the endowment of capital and labor are both 100, do both firms operate?

(d) Suppose $p = 1$. If the endowment of capital and labor are 100 and 400 respectively, do both firms operate?

Problem 2. A small open economy produces two goods, A and B, using two inputs, capital (k) and labor (l). The production function for the two goods are:

$$f_A(l_A, k_A) = \min\{\alpha_A l_A, \beta_A k_A\}$$
$$f_B(l_B, k_B) = \min\{\alpha_B l_B, \beta_B k_B\}$$

Let p_A and p_B be the world output prices of good A and B respectively. Also, let the country's stock of capital and labor be $(K, L) \gg 0$. Denote the prices of capital and labor by r and w respectively. Suppose $K/L \in (1/4, 1/2)$.

(a) Let $p_A = \alpha_A = \alpha_B = 1$ and $\beta_A = 4$, $\beta_B = 2$. Suppose we are in the situation where both A and B are produced in positive quantities. Solve for the competitive equilibrium when both wage rate and rental rate are positive.

(b) Suppose the economy specified above is in a diversified competitive equilibrium with $w, r > 0$. What can you say regarding the factor intensity of industry A compared to industry B? What is the effect of an increase in p_B on the equilibrium input prices?