ECON 6100 4/2/2021

Section 7

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1 Review

An integral part of any equilibrium in an economy is that the produced goods get traded (exchanged) among the consumers. Today we will talk about two simple models which each describe one aspect of this.

1.1 Walrasian Model

Suppose that an economy has consumers $i \in \mathcal{I} \equiv \{1, \cdots, n\}$ and L goods indexed by $l \in \mathcal{L} \equiv \{1, \cdots, L\}$. $x \in \mathbb{R}^L_+$ denotes a goods bundle. There is no production within this economy. Instead, consumers are endowed with bundles $w^i \in \mathbb{R}^L_+$. Consumers also have preferences denoted by utility function $u^i : \mathbb{R}^L_+ \to \mathbb{R}$. So, a Walrasian exchange economy can be characterized as $\mathcal{E} = \left(u^i, w^i\right)_{i\in\mathcal{I}}$.

Given any price vector p, an allocation $(x^i)_{i\in\mathcal{I}}$ is said to be *feasible* if

$$p \cdot x^i \leq p \cdot w^i, \ \forall i \in \mathcal{I} \tag{1}$$

1.1.1 Walrasian Equilibrium

An equilibrium for economy $\mathcal{E}$ is a vector of prices and commodity consumption bundles $\left(p, (x^i)_{i\in\mathcal{I}}\right)$ that satisfies the followings:

1. The good bundle is utility maximizing for each $i \in \mathcal{I}$:

$$x^i \in \arg \max_{p\cdot z \leq p \cdot w^i} u^i(z) \tag{2}$$

Marshallian demand.

2. Market clearing:

$$\sum_{i\in\mathcal{I}} x^i = \sum_{i\in\mathcal{I}} w^i \tag{3}$$

1.1.2 Pareto Optimality

An allocation $(x^i)_{i\in\mathcal{I}} \in \mathbb{R}^{L\cdot}_+$ is said to be Pareto Optimal in economy $\mathcal{E}$ if there exists no feasible $(\hat{x}^i)_{i\in\mathcal{I}}$ such that

- for every $i \in \mathcal{I}$, $u^i(\hat{x}^i) \geq u^i(x^i)$
- for some i, $u^i(\hat{x}^i) > u^i(x^i)$.

1.1.3 Maintained Assumptions

* These assumptions presented here are a stronger than required but makes for simple exposition.

1. For all $i \in \mathcal{I}$, u^i is continuous, increasing and concave
2. For all $i \in \mathcal{I}$, $w^i \gg 0$.

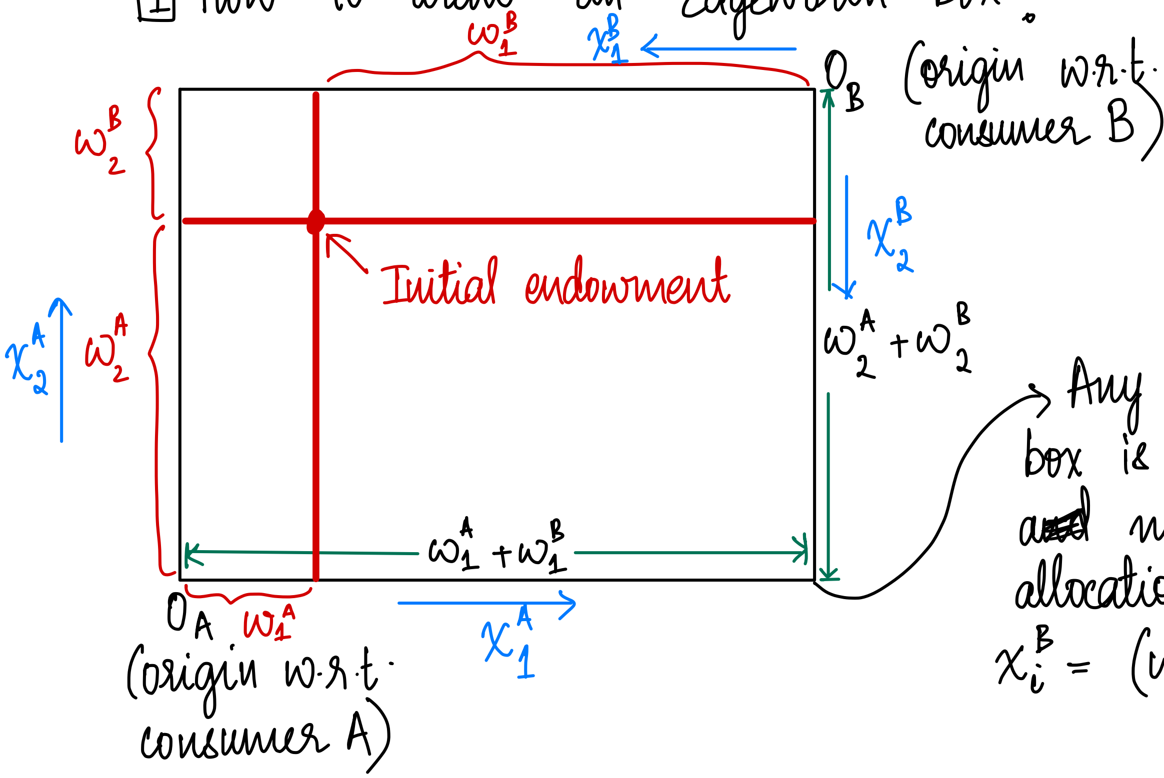
1.1.4 Edgeworth boxes

Turns out that for $I = L = 2$, there is a super cool graphical illustration.

$\mathcal{E} = \{u^A, u^B, \underset{2\times1}{\omega^A}, \underset{2\times1}{\omega^B}\}$

$(0,0)$ for B is $(\omega_1^A+\omega_1^B, \omega_2^A+\omega_2^B)$ for A.

$\boxed{1}$ How to draw an Edgeworth Box?



Any point in this box is a ~~feasible~~ ~~and~~ non-wasteful allocation of resources.

$x_i^B = (\omega_i^A + \omega_i^B) - x_i^A$

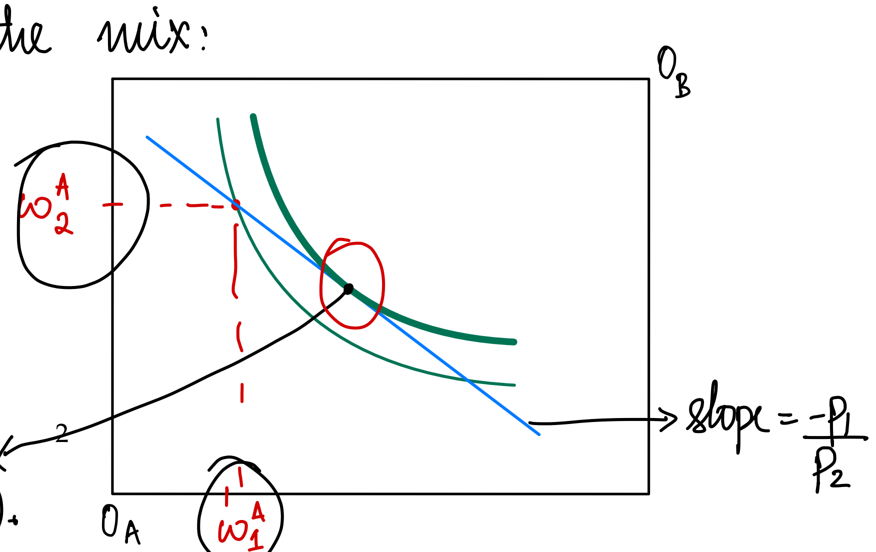
$\boxed{2}$ Adding preferences to the mix:

$u_A(x_1, x_2) = \sqrt{x_1 x_2}$

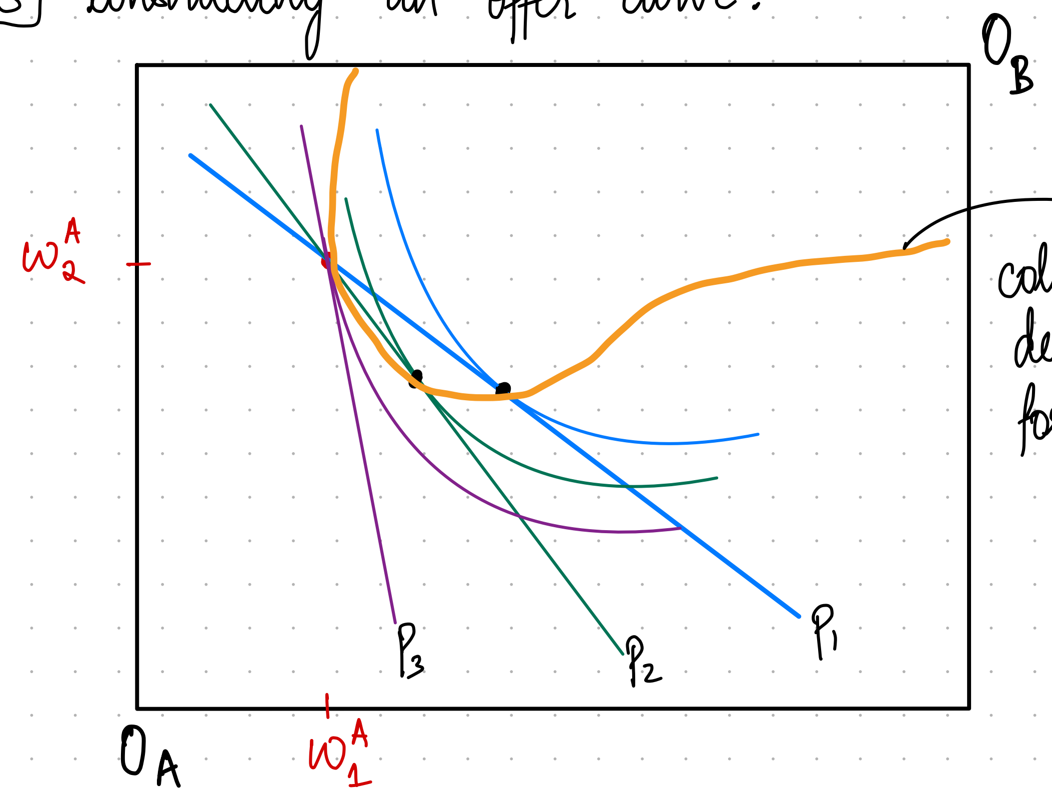
Fix a price vector:

(P_1, P_2)

Marshallian demand (x_1^A, x_2^A) given (P_1, P_2) & (ω_1^A, ω_2^A).



3 Constructing an offer curve:



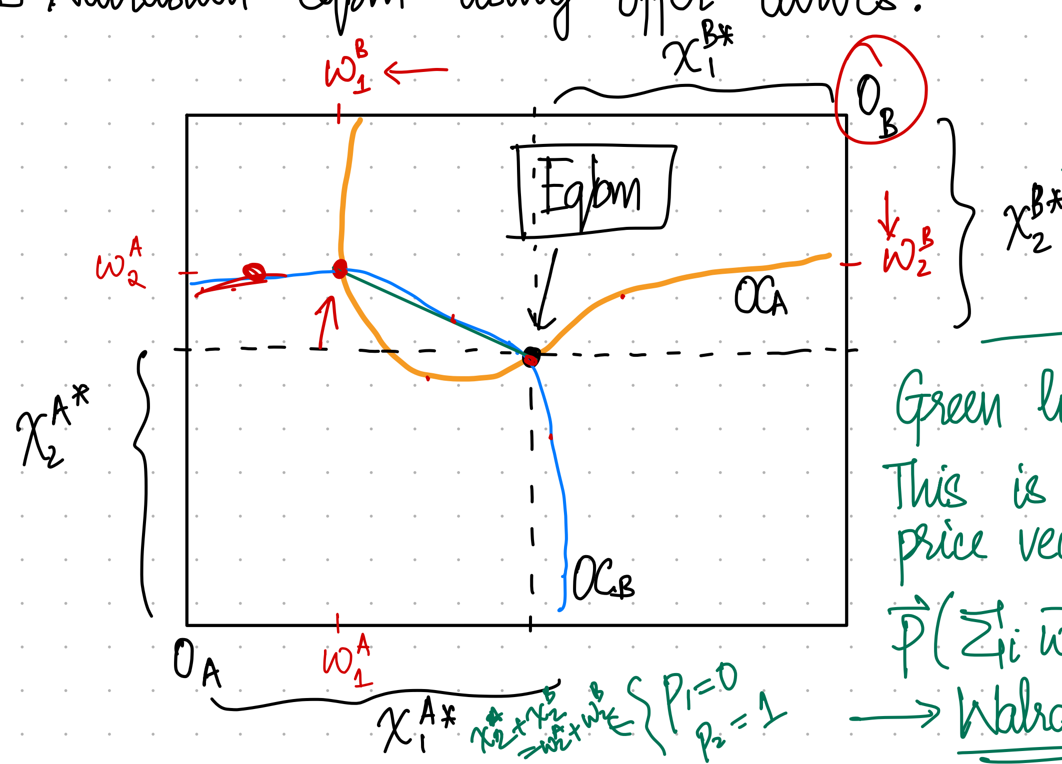
Offer curve: collection of Marshallian demands given $\vec{w}^A$ for each $\vec{p}$.

(*) We are well on our way to computing Walrasian equilibrium. Recall it requires:

i- Allocations are Marshallian demand.

ii- Allocations are feasible. 0 excess demand.

4 Walrasian eqbm using offer curves:



$px \leq pw$

$\downarrow$

w

$px^A = pw^A$

$px^B = pw^B$

$p(x^A + x^B - w^A + w^B) = 0$

Green line slope: $-P_1/P_2$. This is the equilibrium price vector.

$\vec{p}\left(\sum_i \vec{w}^i\right) = \vec{p}\left(\sum_i \vec{x}^i\right)$

$x_2^A + x_2^B = w_2^A + w_2^B \;\{ P_1 = 0,\; P_2 = 1$

$\longrightarrow$ Walras law!

+ Note that OC_A and OC_B pass through the endowment.

$\Rightarrow \exists$ some prices (different for A, B) for which the consumer wants to hang on to its endowment.

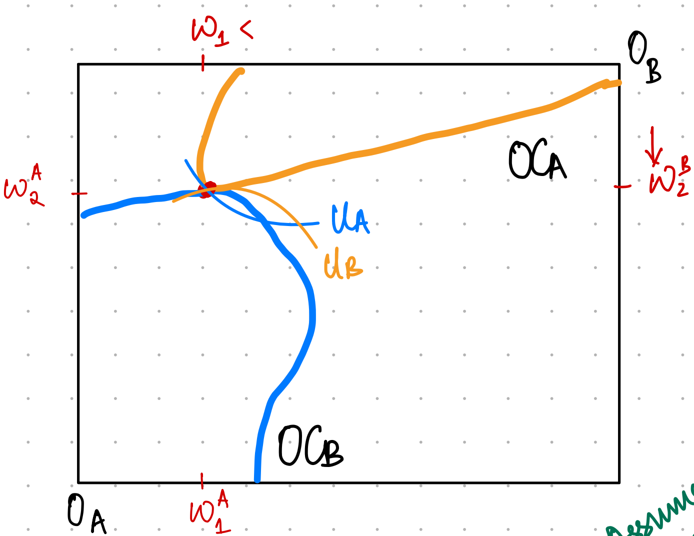
+ For an eqbm to exist, either needs to hold:

i - The same price makes each consumer want to hold on to their endowment

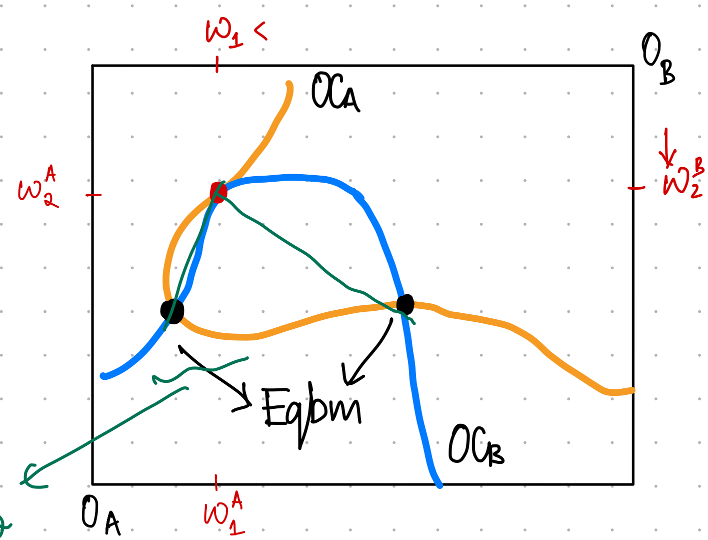
ii - OC_s intersect in another point.

This will solve: $\vec{p} \cdot \sum_i \vec{w}^i = \vec{p} \cdot \sum_i \vec{x}^i$.

$\boxed{5}$ Non-existance and multiple equilibria:



⊛



Assume they are vertically separated

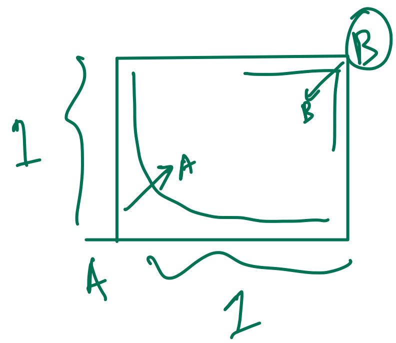
⊛ Each eqbm is supported by a diff. price vector.

Problem 1 (Varian 17.4)**.** There are two consumers A and B with the following utility functions and endowments:

$$u^A(x,y) = \alpha \ln x + (1-\alpha)\ln y \qquad \omega^A = (0,1)$$
$$u^B(x,y) = \min\{x,y\} \qquad \omega^B = (1,0)$$

(a) Illustrate this situation in an Edgeworth box diagram for $\alpha = 1/2$. What are the market clearing prices and the equilibrium allocation.

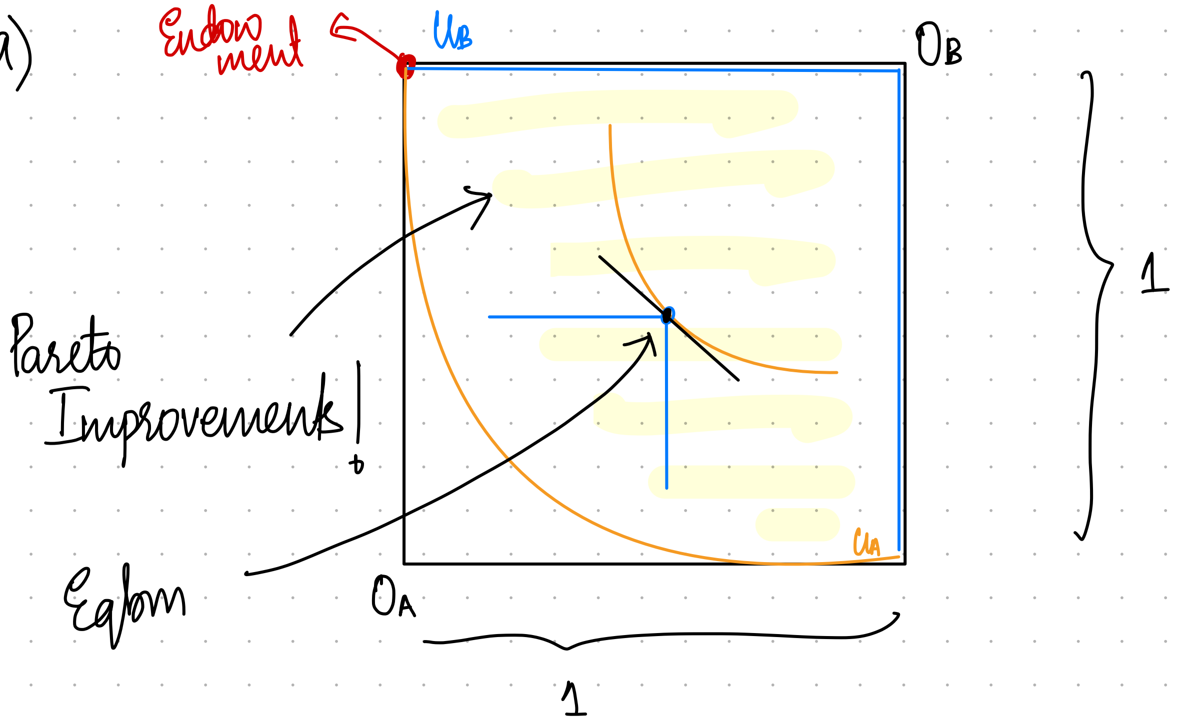
(b) For $\alpha \in (0,1)$, calculate the market clearing prices and the equilibrium allocation. How do things change with α?



$\alpha \uparrow \longrightarrow$ A like x^A more than x^B.

$\Rightarrow p \downarrow$

(a)



$\frac{p_x x}{I} = \alpha$

$I = p$

share of income $= \alpha$ on 1

Share(2) $= 1-\alpha$.

Marshallian demands: (Normalize $p_x = 1$, $p_y = p$) ✓

$x^A = \alpha \cdot p \quad ; \quad p y^A = (1-\alpha)p \longrightarrow y^A = 1-\alpha.$

$p > 0$

$x^A = \frac{p}{2} \quad ; \quad y^A = 1/2.$

$x^B = y^B \quad ; \quad (x^B + p y^B = 1)$

Non-wastefullness: $x^A + x^B = 1 \; ; \; y^A + y^B = 1.$

$\therefore \; CE = \{x^A = x^B = y^A = y^B = 1/2 \; ; \; p = 1\}$ ✓

(b) $y^A = 1-\alpha \Rightarrow y^B = \alpha \Rightarrow x^B = \alpha \Rightarrow x^A = 1-\alpha.$

Using $x^A = \alpha p \Rightarrow p = \frac{1-\alpha}{\alpha}.$

$p_x = 1$

$p_y = p$

Why not? $p_x = 0$ or $p_y = 0$

$CE = \left\{ x^A = x^B = 1-\alpha \; ; \; y^A = y^B = \alpha \; ; \; p = \frac{1-\alpha}{\alpha} \right\}.$

$\longrightarrow$ Discuss

$\omega^A = (0,1)$, $\omega^B = (1,0)$

Problem 2. In a two-person, two good exchange economy, individuals A and B have the following preferences

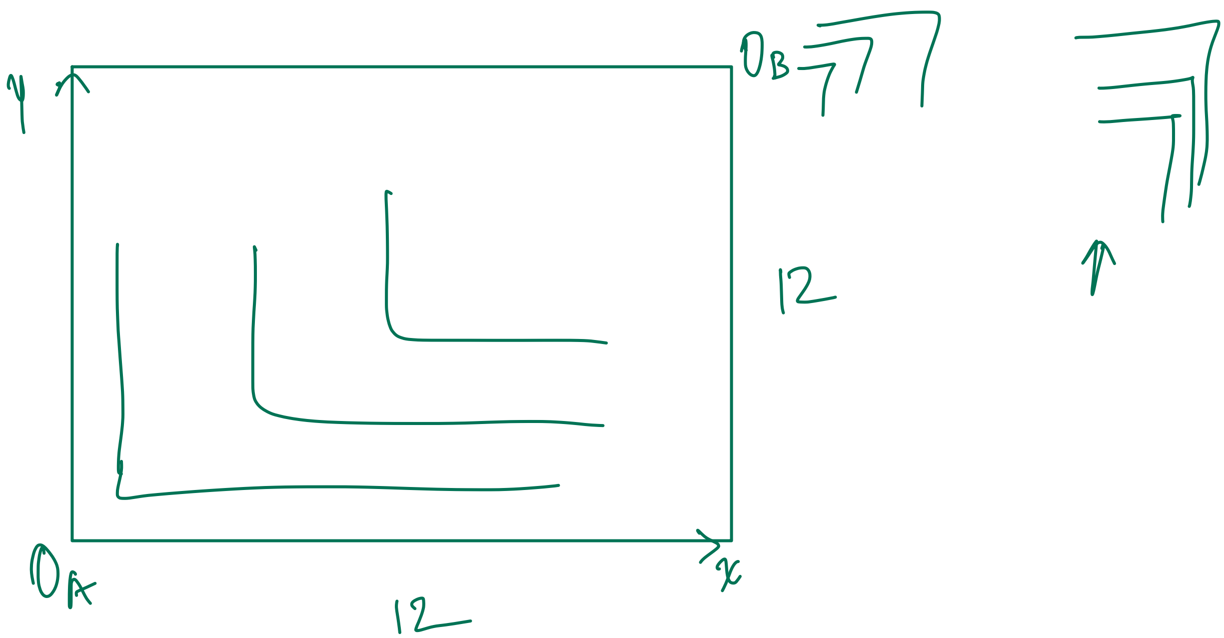
$$u^A(x,y) = \min\{x, 2y\}$$
$$u^B(x,y) = \min\{3x, y\}$$

where x and y are two goods. The aggregate endowment is $(12, 12)$. Answers the following using an Edgeworth box.

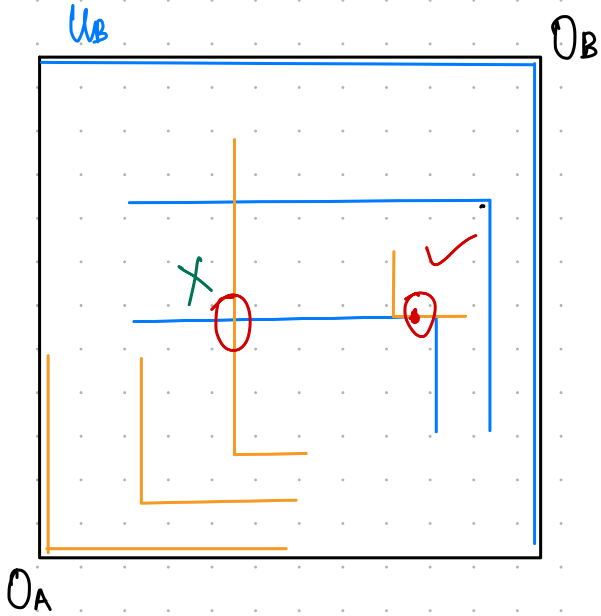
(a) Find the set of Pareto optima.

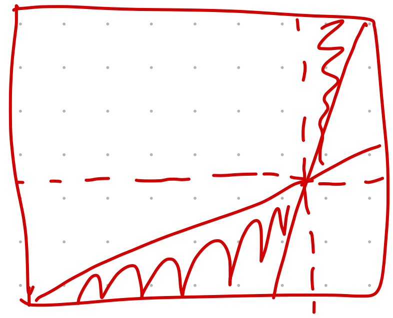
(b) Assume A has all of good x and B all of good y. Describe the competitive equilibria.

(c) Which endowment allocations have competitive equilibria with positive prices for both goods?



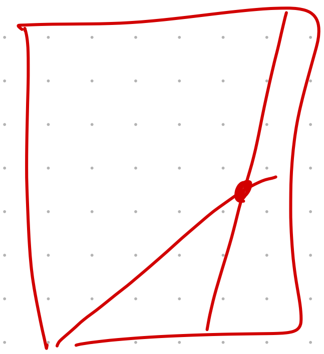
(a)





(b) $\omega_x^A = 12$, $\omega_y^B = 12$

$$\left\{ \begin{array}{l} x_A = 2y_A \\ 3x_B = y_B \end{array} \right\}^*$$



Solve it graphically. $\longrightarrow$ HOW?

$$(x^A, y^A) = \left(\frac{48}{5}, \frac{48}{10}\right) \qquad (x^B, y^B) = (3.4, 7.2)$$

9.6 4.8

$p = 1/2$.

(c) <u>Hint:</u> What would Walras's Law say?

eg: check $w^A = (8,0)$, $w^B = (4,12)$.

with $P_x > 0$, $P_y = 0$

$$\max_{c,S,L} u(c,S)$$

$$\text{s.t. } \left.\begin{array}{l} c \le F(L) \\ S \le 24-L \end{array}\right\} \text{binding in eqbm because } u(\cdot) \text{ is LNS.}$$

$$c = F(L)$$
$$S = 24-L$$

1.2 Crusoe Economy

The aim here is to study the decisions of an agents who is both a producer and a consumer. Crusoe is endowed with 24 hours each day which (s)he may allocate to leisure (chilling on an island) or harvesting coconuts. Suppose L is the number of hours Crusoe allocates to harvesting coconuts, the number of coconuts (s)he harvests is given by production technology, $F(\cdot)$:

$$y = F(L) \tag{4}$$

Since we are economists, we assume that F is C^2, concave and satisfies the Inada conditions.

We will use c to denote the number of coconuts Crusoe consumes and $S = 24 - L$ the leisure. Crusoe's preference over coconuts c and leisure S may be represented by utility function $u(c, S)$ which is assumed to be C^2, increasing and concave.

Problem 3. In the Crusoe economy, characterize the optimal *centralized* allocation of time to coconut harvesting L and leisure S. That is, if Crusoe was to decide for himself how much to eat and how much to chill given his preferences, how would he do so?

$$\max_{L} u(F(L), \underbrace{24-L}_{S})$$

$$\text{FOC: } \frac{d}{dL} u(F(L), 24-L) = 0$$

$$\frac{d(24-L)}{dL} = -1$$

$$u'_c \cdot F'(L) - u'_S = 0$$

$$q = y$$

$$\therefore \frac{u'_S}{u'_c} = F'(L) = -\frac{dy}{dS}$$

$$\frac{MU_1}{MU_2}$$

$$MRS(S,C) = -\frac{dc}{dS} = \frac{u'_S}{u'_c} = -\frac{dq}{dS} = F' = MRT_{S,C}.$$

Producer $\longrightarrow$ Crusoe $\longleftarrow$ Cons.

π max $\quad$ Umax

$\hookrightarrow p \cdot x \leq m$

Problem 4. Suddenly, one day Crusoe reads Hayek's work and decides that he wants to try a *decentralized* allocation. In this model with 2 commodities where the price of labor relative to coconuts is w (and Crusoe is a price taker), figure out the decentralized allocation.

Profits: $\pi = F(L) - wL$

$$\frac{d\pi}{dL} = 0 = F'(L) - w \Rightarrow \boxed{w = F'(L)}$$

Budget: $Y = \pi + \underline{24w} = q - wL + 24w$

$$Y = q + wS = c + wS$$

Crusoe spends on c, S

$c \to = 1$, $S \to w$

$$\underbrace{S = \frac{Y-c}{w}}$$

Consumer:

$$\max u\left(c, \frac{Y-c}{w}\right)$$

$$\frac{du}{dc} = u'_c - \frac{1}{w} u'_S = 0$$

MRS (S,c)

$$\boxed{\frac{u'_S}{u'_c} = w = F'(L)}$$

$=$ MRT (S,c)

$+ \quad c = q \;;\; L = 24 - S$