ECON 6100 4/9/2021

Section 8

Lecturer: Larry Blume *TA: Abhi Ananth*

*Developed from Fikri Pitsuwan's material.

Problem 1 (2005 Aug III)**.** There are three agents in the economy A, B, and C. There are three goods in the economy (x_1, x_2, x_3). Agent A has 1 unit of x_1, agent B has $b \in [1, 2)$ units of x_2 and agent C has 1 unit of x_3. The utility functions of the agents are

$$u^A(x_1, x_2, x_3) = \min\{x_1, x_2\}$$
$$u^B(x_1, x_2, x_3) = \min\{x_2, x_3\}$$
$$u^C(x_1, x_2, x_3) = \min\{x_1, x_3\}$$

Let p_1, p_2, and p_3 denote the prices of goods.

(a) In a CE, can all prices be positive? What happens when 2 or all prices are 0?

(b) Write down the excess demand function of each good.

(c) If $p_3 = 1$, find the other prices.

(d) Suppose $p_3 = 1$, then how will each agent's utility change with a change in b?

Problem 2 (2001 June IV). Consider a private ownership economy with two individuals, Mr. 1 and Mr. 2 and two goods x and y. Consumers cannot consume in negative quantities. Mr. 1 and Mr. 2 has the following utility functions

$$u^1 = x_1 - \gamma y_2$$
$$u^2 = (x_2 y_2)^{1/2}$$

where $\gamma \in [0, 1)$. Each consumer has endowment of 1 unit of each good. Let good x be the numeraire good and denote the price of good y by p.

(a) Find the CE allocation and price for this economy

(b) For what values of γ is the CE Pareto optimal?

(c) Can a sales tax τ on good y (amount collected from tax is given to Mr. 1 as lump-sum) be constructed such that all CE are PO?

Problem 3. Consider an exchange economy with L goods and N consumers. Each consumer's utility function is of the form $u_n(x_1, x_2, \dots, x_L) = \sum_l v_n(x_l)$, where each v_n is strictly concave, strictly increasing, differentiable and satisfies Inada condition at the origin. Suppose that each consumer has a strictly positive endowment $w_n = (w_{n1}, w_{n2}, \dots, w_{nL}) \gg 0$.

(a) Show that if $\sum_n w_{n1} = \sum_n w_{n2} = \cdots = \sum_n w_{nL}$, then the economy has at most one equilibrium.

(b) Show that if $\sum_n w_{n1} > \sum_n w_{n2} > \cdots > \sum_n w_{nL}$, then for the competitive equilibrium price vector p^*, $p_1^* < p_2^* < \cdots < p_L^*$.