ECON 6100 4/9/2021

Section 8

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*Developed from Fikri Pitsuwan's material.

Problem 1 (2005 Aug III)**.** There are three agents in the economy A, B, and C. There are three goods in the economy (x_1, x_2, x_3). Agent A has 1 unit of x_1, agent B has $b \in [1, 2)$ units of x_2 and agent C has 1 unit of x_3. The utility functions of the agents are

$$u^A(x_1, x_2, x_3) = \min\{x_1, x_2\}$$
$$u^B(x_1, x_2, x_3) = \min\{x_2, x_3\} \rightarrow x_2^{B*} = x_3^{B*}$$
$$u^C(x_1, x_2, x_3) = \min\{x_1, x_3\}$$

Let p_1, p_2, and p_3 denote the prices of goods.

(a) In a CE, can all prices be positive? What happens when 2 or all prices are 0?

(b) Write down the excess demand function of each good.

(c) If $p_3 = 1$, find the other prices.

(d) Suppose $p_3 = 1$, then how will each agent's utility change with a change in b?

In any CE: i- Each agent consumes Marshallian demand.

ii- $\sum_i x_i = \sum_i \omega_i \leftarrow$ market clearing.

(a) What if $b = 1$? Assuming $P_1, P_2, P_3 > 0$

Marshallian: $x_1^A = x_2^A$; $P_1 x_1^A + P_2 x_2^A = P_1$; $x_3^A = 0$

$x_2^B = x_3^B$; $P_2 x_2^B + P_3 x_3^B = P_2 b$; $x_1^B = 0$

$x_1^C = x_3^C$; $P_3 x_3^C + P_1 x_1^C = P_3$; $x_2^C = 0$

MC: $x_1^A + x_1^C = 1$

$\rightarrow x_2^A + x_2^B = 1\ b$

$x_3^A + x_3^C = 1$.

$P_1 = P_2 = P_3 = 1$.

$x_1^A = 1/2$ $x_2^B = 1/2$ $x_3^C = 1/2$

$b\uparrow \rightarrow b>1$?

Hint: Substitute Marshallian demand in MC.

$$x_2^A + x_2^B = x_3^B + x_1^A$$

$$\hookrightarrow 1 - \underbrace{x_1^c}_{1-x_3^c}$$

$$= 1 \neq b.$$

If $\vec{p} = \vec{0}$:

A: $\max\limits_{x_1^A, x_2^A, x_3^A} \min\{x_1, x_2\}$ s.t. $\vec{p}\cdot\vec{x}^A \leq \vec{p}\cdot\vec{\omega}^A$

$0 \leq 0$ ✓

Unbounded demand! Violates MC.

If $p_1, p_2 = 0$:

A: $\max \min\{x_1, x_2\}$.

s.t. $0 + p_3 \cdot x_3^A \leq 0$. ✓

$\forall x_1, x_2 \geq 0$ & $x_3 = 0$.

(b)

Marshallian: $x_1^A = x_2^A$; $p_1 x_1^A + p_2 x_2^A = p_1$; $x_3^A = 0$

$x_2^B = x_3^B$; $p_2 x_2^B + p_3 x_3^B = p_2 b$; $x_1^B = 0$

$x_1^c = x_3^c$; $p_3 x_3^c + p_1 x_1^c = p_3$; $x_2^c = 0$

MC: $x_1^A + x_1^c = 1$

$\rightarrow x_2^A + x_2^B = b$

$x_3^A + x_3^C = 1.$

$$x_1^A = \frac{p_1}{p_1 + p_2} = x_2^A$$

$$x_2^B = \frac{b p_2}{p_2 + p_3} = x_3^B$$

Errata Corrige:

Suppose:

Marshallian:

$$x_1^A = x_2^A \;;\; p_1 x_1^A + p_2 x_2^A = p_1 \;;\; x_3^A = 0$$

$$x_2^B = x_3^B \;;\; p_2 x_2^B + p_3 x_3^B = p_2 b \;;\; x_1^B = 0$$

$$x_1^C = x_3^C \;;\; p_3 x_3^C + p_1 x_1^C = p_3 \;;\; x_2^C = 0$$

MC:

$$x_1^A + x_1^C = 1$$

$$x_2^A + x_2^B = b$$

$$x_3^A + x_3^C = 1.$$

$$\to \quad x_1^A = x_2^A = \frac{p_1}{p_1+p_2} \;;\; x_3^A = 0$$

$$x_2^B = x_3^B = \frac{p_2 b}{p_2+p_3} \;;\; x_1^B = 0$$

$$x_3^C = x_1^C = \frac{p_3}{p_1+p_3} \;;\; x_2^C = 0$$

$$\frac{\frac{2-b}{b}}{\frac{b+2-b}{b}}$$

MC:

$$\left.\begin{aligned} \frac{p_1}{p_1+p_2} + \frac{p_3}{p_1+p_3} &= 1 \\ \frac{p_1}{p_1+p_3} + \frac{p_2 b}{p_2+p_3} &= b \\ \frac{p_2 b}{p_2+p_3} + \frac{p_3}{p_1+p_3} &= 1 \end{aligned}\right\}$$

$$p_1 = 1$$

$$p_2 = p_3 = \frac{2-b}{b}$$

Equilibrium exist where Marsh. Dem + MC are satisfied.

$$x_1^A = x_2^A = b/2 \qquad x_3^C = x_1^C = \frac{2-b}{2}$$

$$x_2^B = x_3^B = b/2$$

* Notice that this satisfied both

Marshallian demand conditions

&

Market clearing.

My error earlier was caused by substituting

$\boxed{x_3^C = x_2^C}$ & $x_3^B = x_2^B$

not true

$x_3^C = x_1^C$

into $x_2^B + x_2^C = b$.

which gave $x_3^B + x_3^C = b > 1$!

TL;DR: my brain doesn't work on Fridays.

Happy Weekend!

* Thanks for the Q Hyewon!

$$Z_1(p,\omega) = x_1^A + x_1^C - 1$$

$$\stackrel{!}{=} \underbrace{\frac{p_1}{p_1+p_2}}_{x_1^A} + \frac{p_3}{p_1+p_3} - 1$$

$$Z_2 = \frac{p_1}{p_1+p_2} + \frac{bp_2}{p_2+p_3} - b$$

$$Z_3 = \frac{bp_2}{p_2+p_3} + \frac{p_3}{p_1+p_3} - 1.$$

(c) What happens when $p_3 = 1$? In eqbm $Z_1 = Z_2 = Z_3 = 0$.

$$Z_1 - Z_2 = 0 \rightarrow \frac{1}{p_1+1} - \cancel{\frac{p_2 b}{p_2+1}} = 1-b$$

$$Z_3 = 0 \rightarrow \cancel{\frac{p_2 b}{p_2+1}} + \frac{1}{p_1+1} = 1.$$

$(+)$

$p_2 = 1, p_3 = 1$

Using Walras Law $\vec{p} \cdot \vec{Z} = 0$.

$$\rightarrow p_1 = \frac{b}{2-b} \quad ; \quad p_3 = 1 ; p_2 = 0$$

(d) Fix $p_3 = 1$; how does x^A change with b?

$$x_1^A = x_2^A = \frac{p_1}{p_1+p_2} = b/2$$

As $b \uparrow \longrightarrow x^A \uparrow$.

$$x_3^A = 1 - x_1^A \longrightarrow 1 - b/2.$$

Problem 2 (2001 June IV)**.** Consider a private ownership economy with two individuals, Mr. 1 and Mr. 2 and two goods x and y. Consumers cannot consume in negative quantities. Mr. 1 and Mr. 2 has the following utility functions

$$u^1 = x_1 - \gamma y_2$$
$$u^2 = (x_2 y_2)^{1/2}$$

Mr1: $\max_{x_1, y_1} u$ s.t. B.C.

$\boxed{\sum^I x = \sum \omega_x}$ X Not what Mr1 looks at when choosing x_1, y_1.

where $\gamma \in [0, 1)$. Each consumer has endowment of 1 unit of each good. Let good x be the numeraire good and denote the price of good y by p.

(a) Find the CE allocation and price for this economy

(b) For what values of γ is the CE Pareto optimal?

(c) Can a sales tax τ on good y (amount collected from tax is given to Mr. 1 as lump-sum) be constructed such that all CE are PO?

(a) Mr 1:

$$\max_{x_1, y_1} \; x_1 - \gamma y_2 \qquad y_2 = 1+1-y_1$$
$$\text{s.t.} \; x_1 + p y_1 \leq 1+p$$

$$x_1^* = \cancel{0} \; \cancel{\frac{1+p}{}} \; 1+p$$
$$y_1^* = \cancel{\frac{1+p}{p}} \; 0$$

Mr 2:

$$\max \sqrt{x_2 y_2}$$
$$\text{s.t.} \; x_2 + p y_2 \leq 1+p.$$

$$x_2 = \frac{1}{2}(1+p)$$

$$x_2^* = \frac{1+p}{2}$$
$$y_2^* = \frac{1+p}{2p}$$

MC: (x) $1+p + \frac{1+p}{2} = 2. \rightarrow p^* = 1/3$

(y)

In CE: $x_1^* = 4/3$, $y_1^* = 0$

$(x_2^* = 2/3, \; y_2^* = 2) \rightarrow \bar{u}$

(b) $(x_1^*, x_2^*, y_1^*, y_2^*)$ is P.O. if $\exists \bar{u}$:

$$\begin{cases} \max\limits_{x_1,x_2,y_1,y_2} & x_1 - \gamma y_2 \\ \text{s.t.} & \sqrt{x_2 y_2} \geq \bar{u} \\ & x_1 + x_2 \leq 2 \\ & y_1 + y_2 \leq 2 \end{cases}$$

$\longleftrightarrow$ binding. + Non-neg.

S: $\underline{\gamma \leq p}$:

$$\max \ \underbrace{2 - x_2}_{} - \gamma y_2 \quad \text{s.t.} \quad \sqrt{x_2 y_2} \geq \bar{u}$$

$$0 \leq x_2 \leq 2$$

$$0 \leq y_2 \leq 2.$$

$$\mathcal{L} = 2 - x_2 - \gamma y_2 + \lambda_1 \left(\sqrt{x_2 y_2} - \bar{u}\right) + \lambda_2 x_2 + \lambda_3 (2 - x_2)$$

$$+ \lambda_4 y_2 + \lambda_5 (2 - y_2).$$

Plug in CE from (a).

$$\lambda_2 = \lambda_1 = \lambda_3 = \lambda_4 = 0.$$

$$\lambda_5 \geq 0 \quad (y^2 = 2).$$

$$(\lambda_5 = p - \gamma) \geq 0 \quad \rightarrow \text{From solving FOC } (x_2 \ \& \ y_2).$$

$$\underline{\gamma \leq p}$$

(c) $\text{Mr } 1:$ $\max \; x_1 - \gamma y_2$

$$\text{s.t.} \quad x_1 + (p+\tau) y_1 \leq 1 + p + \tau(y_1 + y_2)$$

$\text{Mr}:$ $\max \; \sqrt{x_2 y_2}$

$$\text{s.t.} \quad x_2 + (p+\tau) y_2 \leq 1 + p$$

→ CE in terms of τ.

→ PO conditions & check τ that makes CE = PO.

Problem 3. Consider an exchange economy with L goods and N consumers. Each consumer's utility function is of the form $u_n(x_1, x_2, \ldots, x_L) = \sum_l v_n(x_l)$, where each v_n is strictly concave, strictly increasing, differentiable and satisfies Inada condition at the origin. Suppose that each consumer has a strictly positive endowment $w_n = (w_{n1}, w_{n2}, \ldots, w_{nL}) \gg 0$.

(a) Show that if $\sum_n w_{n1} = \sum_n w_{n2} = \cdots = \sum_n w_{nL}$, then the economy has at most one equilibrium.

(b) Show that if $\sum_n w_{n1} > \sum_n w_{n2} > \cdots > \sum_n w_{nL}$, then for the competitive equilibrium price vector p^*, $p_1^* < p_2^* < \cdots < p_L^*$.

(a) $\cancel{\vec{\omega}_n \neq \vec{\omega}_m \ \forall n \neq m}$

Consumer n's problem: $\max_{\vec{x}_n} \sum_l v_n(x_{n,l})$

s.t. $\vec{p} \cdot \vec{x}_n \leq \vec{p} \cdot \vec{\omega}_n$

FOC: w.r.t. good l: $v_n'(x_{n,l}) = \lambda_n p_l$.

" " k: $v_n'(x_{n,k}) = \lambda_n p_k$.

v is ↑ & concave (str.)

If $p_l > p_k \implies v_n'(x_{n,l}) > v_n'(x_{n,k})$

$\implies x_{n,l} < x_{n,k}$ ($\because$ Str. concave) $\forall n$.

$\sum_n x_{n,l} < \sum_n x_{n,k}$.

$\sum_n \omega_{n,l} = \sum_n \omega_{n,k}$.

|| Violation of MC.

In all eqbm → prices are equal ⟹ same demand.

(b) MC: $\sum_n^I \omega_{n,l} > \sum_n^I \omega_{n,k}$.

$\quad\quad\quad\quad \parallel \quad\quad\quad\quad \parallel$

$\sum_n^I x_{n,l} > \sum_n^I x_{n,k}$

(TS) $p_l < p_k$

Tow. a Contradiction: $p_l \geq p_k$. $\Rightarrow x_{n,l} \leq x_{n,k}$, $\forall n$

$\sum^I x_{n,l} \leq \sum^I x_{n,k}$

leads to cont.