ECON 6100		4/16/2021
	Section 9	
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Problem 1. (2013 June 5) Consider an economy with one consumer. The consumer owns one unit of good X. There is a technology that turns good X into good Y. The production function is $y = x^2$, where y is the output of good Y that can be produced with x units of good X. Another technology turns good X and good Y into good Z. Its production function is $z = x^\alpha y^\beta$, where z is the output of good Z that can be produced with x units of good X and y units of good Y. The consumer cares only about consumption of good Z, more is better. Consider two different managerial regimes for the two technologies. In regime A, each technology is owned by a separate profit-maximizing firm. In regime B a single firm owns both technologies and manages them jointly. Profits, if any, go to the consumer.

(a) Discuss $\boxed{\text{equilibrium}}$ and optimality for regime A. Prove any claims you state.

(b) Discuss equilibrium and optimality for regime B. Prove any claims you state.

Optimality ? Eqbm ?

(a) Optimal: Socially

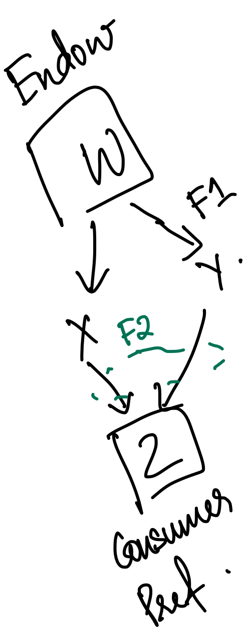
Preference $u(x,y,z) = z$

$$\max_{x,y,z} \; z \quad \text{s.t.} \quad z \overset{F2}{\le} x^\alpha y^\beta, \quad y \overset{\text{Firm 1}}{\le} x^2, \quad x \le 1, \quad x,y,z \ge 0.$$

Consumer

Binding

z is monotone incr. in z.



$$\max_{x_1,x_2} \; x_1^\alpha x_2^{2\beta} \quad \text{s.t.} \quad x_1 + x_2 \le 1$$

$$\max_{0 \le x_1 \le 1} \; x_1^\alpha (1-x_1)^{2\beta} \longrightarrow \quad x_1^* = \frac{\alpha}{\alpha+2\beta}, \quad x_2^* = \frac{2\beta}{\alpha+2\beta}.$$

Consumer: $\quad P_z, P_Y, P_x = 1$

$$\max\ z \quad \text{s.t.} \quad P_z \cdot z \leq 1 + \Pi_1(\vec{p}) + \Pi_2(\vec{p}).$$

F2:

$$\max\ P_z \cdot x^{\alpha} y^{\beta} - x - P_y \cdot y.$$

F1:

$$\max_{x_0}\ P_y x_0^2 - x_0$$

MC:

$$x_0 + x \leq 1.$$

I.R.S. $\Rightarrow P_y > 0$
$\exists\, \bar{x}_0 : \forall x > \bar{x}_0,$
$\pi > 0.$
$\rightarrow$ No eqbm exist.

(b) Opt:

$$z = x^{\alpha+2\beta}$$

$$\boxed{y = x^2}.$$

$x \xrightarrow{T1} x^2.$
$1 \longrightarrow 1.$
$z\ (0,1) \longrightarrow \underline{0}.$

$$\max\ z \quad \text{s.t.} \quad z \leq \hat{x}^{\alpha}(\tilde{x}^2)^{\beta}$$
$$\hat{x} + \hat{x} \leq 1.$$

Eqbm:

C.P $\longrightarrow$ $\max\ z$

$$\text{s.t.} \quad P_z \cdot z \leq 1 + \Pi_0$$

$$\boxed{z^* = \frac{1+\Pi_0}{P_z}.}$$

(assume $P_z > 0$).

Firm: $\quad \max\ P_z \cdot \hat{x}^{\alpha}(\tilde{x}^2)^{\beta} - 1(\hat{x} + \tilde{x})$

$\alpha + 2\beta \leq 1. \longrightarrow$ Assumed to non IRS.

MC: $\quad \hat{x} + \tilde{x} = 1.$

FOC: $\alpha \hat{x} = 2\beta(1-\tilde{x})$.

$$\tilde{x}^* = \frac{2\beta}{\alpha+2\beta} \quad ; \quad \hat{x}^* = \frac{\alpha}{\alpha+2\beta} \quad //$$

$$\Pi^* = p_2 \cdot (\tilde{x}^*)^{2\beta} (\hat{x}^*)^{\alpha} - 1 .$$

$$p_2 = \frac{1+\Pi^*}{2} .$$

$$p_2 = \frac{\cancel{1} + p_2 \cdot \cancel{\tilde{x}^{2\beta} \hat{x}^{\alpha}} - \cancel{1}}{\cancel{\tilde{x}^{2\beta} \hat{x}^{\alpha}}} = p_2$$

Problem 2. (2013 June 2) Fran and Pat are the only two occupants of an island with adequate land to grow and make cornmeal (C) and grape juice (G), neither of which requires any scarce inputs besides time. Fran and Pat have identical preferences $u(C, G) = \min(C, G/2)$. Using her entire time budget, Fran can produce at most 8kg of cornmeal and no grape juice, or at most 16L of grape juice and no cornmeal, or any linear combination that respects these constraints. Using his entire time budget, Pat can produce at most 3kg of cornmeal and no grape juice, or 12L of grape juice and no cornmeal, or any linear combination that respects these constraints.

(a) Draw the production possibility frontier with cornmeal (C) on the x-axis. Label the axes and all kink points. Individual PPF (Autarky) + Joint.

(b) Who has the comparative advantage in producing cornmeal? Why?

(c) In autarky (no trade between Fran and Pat), what are Fran and Pat's maximal utility values?

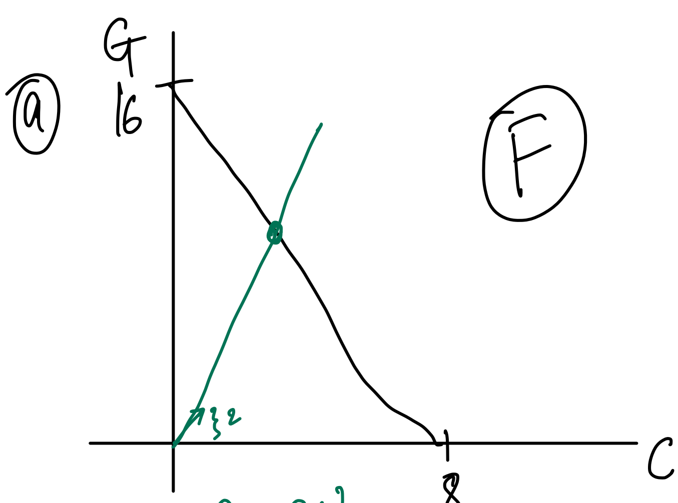
(d) Assuming a competitive equilibrium when trade between Fran and Pat is allowed, what are the total quantities of C and G produced? What is the equilibrium price of grape juice in terms of cornmeal (L/kg)?

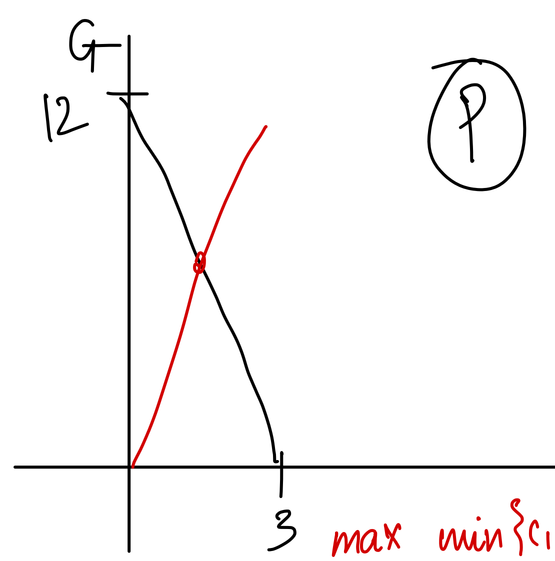
(e) show the consumption expansion path and the competitive equilibrium on the same diagram as your PPF. Label the competitive equilibrium.

(f) In the competitive equilibrium, what are Fran and Pat's maximal utility values?

(g) In units of utility, what are the total gains to trade of the competitive equilibrium as compared to autarky?

(h) Who gains more in the competitive equilibrium, Fran or Pat? By how much?





$\max \min\{c, g/2\}$

$2c + g \le 16$

$2c + 2c \le 16 \rightarrow c^* = 4 ; g = 8^*$

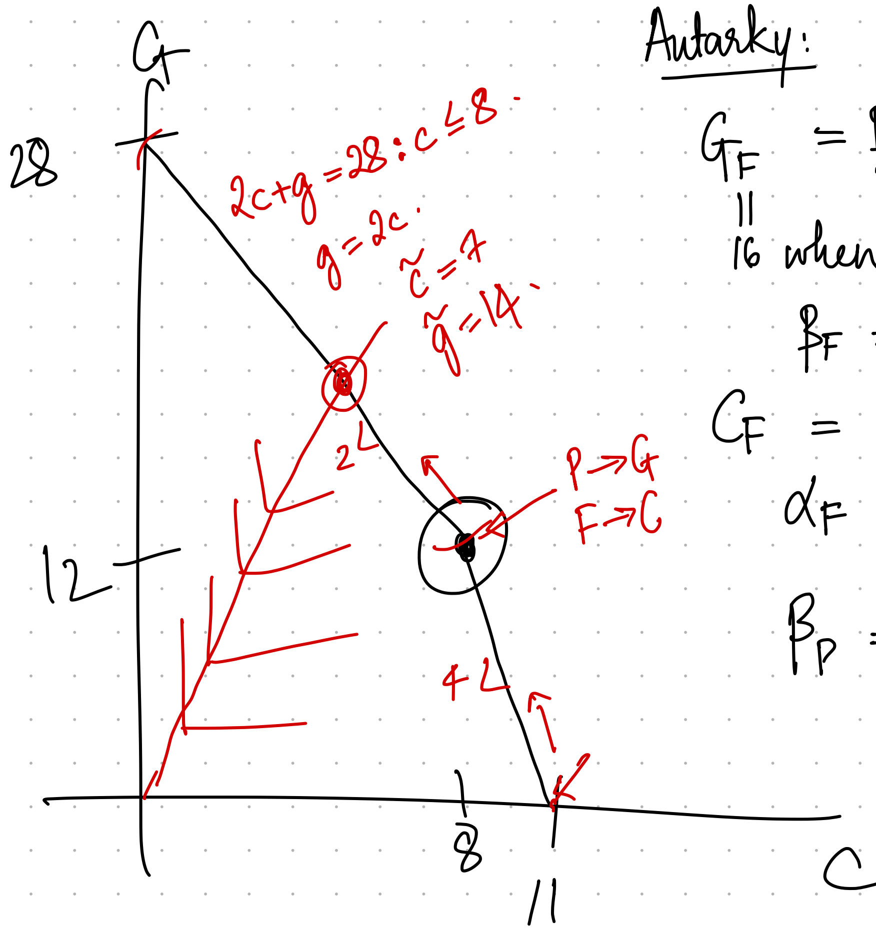
$c = g/2 \rightarrow g = 2c.$

$\max \min\{c, g/2\}$

s.t. $4 \cdot c + g \le 12.$

$c^* = 2, g^* = 4.$

Joint:



Autarky:

$G_F = \beta_F L_F^G \rightarrow 1$.

$\parallel$

16 when $L_L^G = 1$.

$\beta_F = 16$.

$C_F = \alpha_F \cdot L_F^C$

$\alpha_F = 8$

$\beta_P = 12$; $\alpha_P = 3$

Compare $\dfrac{\beta_F}{\alpha_F}$ vs $\dfrac{\beta_P}{\alpha_P}$

Fran $\dfrac{16}{8} = 2$ vs $\dfrac{12}{3} = 4$

Start when all labor is directed to C

$= 8 + 3 = 11$.

Lets' remove labor & direct to G in an efficient way (P produce G).

Kink → F makes C

P makes G.

Apply Ind W. Thm: $\longrightarrow$ CE $\rightarrow$ Opt.

Opt $\rightarrow$ CE I.

$Y \cap \{-Y\} = 0$

$$\max \ \min\{c, g/2\}$$
$$\text{s.t.} \quad PPS$$

$\tilde{c} = 7$; $\tilde{g} = 14$.

$g = 2c$.

In any CE: $MRS = \frac{P_G}{P_c} \rightarrow \frac{P_G}{P_c} = 1/2$.

P

$$\max \ \min\{c, g/2\}$$
$$\text{s.t.} \quad p_c \cdot c + g \cdot p_g \leq p_g \cdot 12.$$

$\begin{bmatrix} \tilde{c} = 7 \\ \tilde{g} = 14 \end{bmatrix}$.

F

$$\max \ \min\{c, g/2\}$$
$$\text{s.t.} \quad p_c \cdot c + g \cdot p_g \leq p_c \cdot 7 + p_g \cdot 2.$$

M.C.

$$c_P + c_F \leq \tilde{c}$$
$$g_P + g_F \leq \tilde{g}$$

Problem 3. (Inspired by June 2012 V) Consider an economy with endowments K^*, L^* and N^* of capital, labor and land, respectively. It can produce food (F) and clothing (C). The production function of food and clothing are

$$F = (N_F)^a (L_F)^{1-a}$$
$$C = (K_C)^b (L_C)^{1-b}$$

~~3x2x1~~

where both $a, b \in (0, 1)$. The economy engages in free trade at world prices for food and clothing given by p and q, respectively.

(a) Assume that the economy is at free-trade competitive equilibrium where it is producing both the goods. Denoting the competitive factor returns of land, labor and capital by s, w and r, respectively, write down the equations that characterizes such an equilibrium.

(b) Suppose the price of clothing q goes up. What happens to w? Draw a diagram to illustrate your answer.

(c) Obtain the effects of an increase in q on the returns to land and capital. Stolper–Samuelson.

(d) Show that the return to labor goes up proportionately less than the price of clothing.

(elasticity)

$$\frac{dw}{w} < \frac{dq}{q}$$

$$\frac{dw}{dq}\frac{q}{w} < 1$$

(a) $\max \ \Pi_F = p\, N_F^a L_F^{1-a} - s N_F - w L_F$

$$p a \left(\frac{L_F}{N_F}\right)^{1-a} = s \quad \text{—(1)}$$

$$p(1-a)\left(\frac{N_F}{L_F \uparrow}\right)^{a} = w \downarrow \quad \text{—(2)}$$

$\max \ \Pi_C = q\, K_C L_C^{1-b} - w L_C - r K_C$

$$q b \left(\frac{L_C}{K_C}\right)^{1-b} = r \quad \text{—(3)}$$

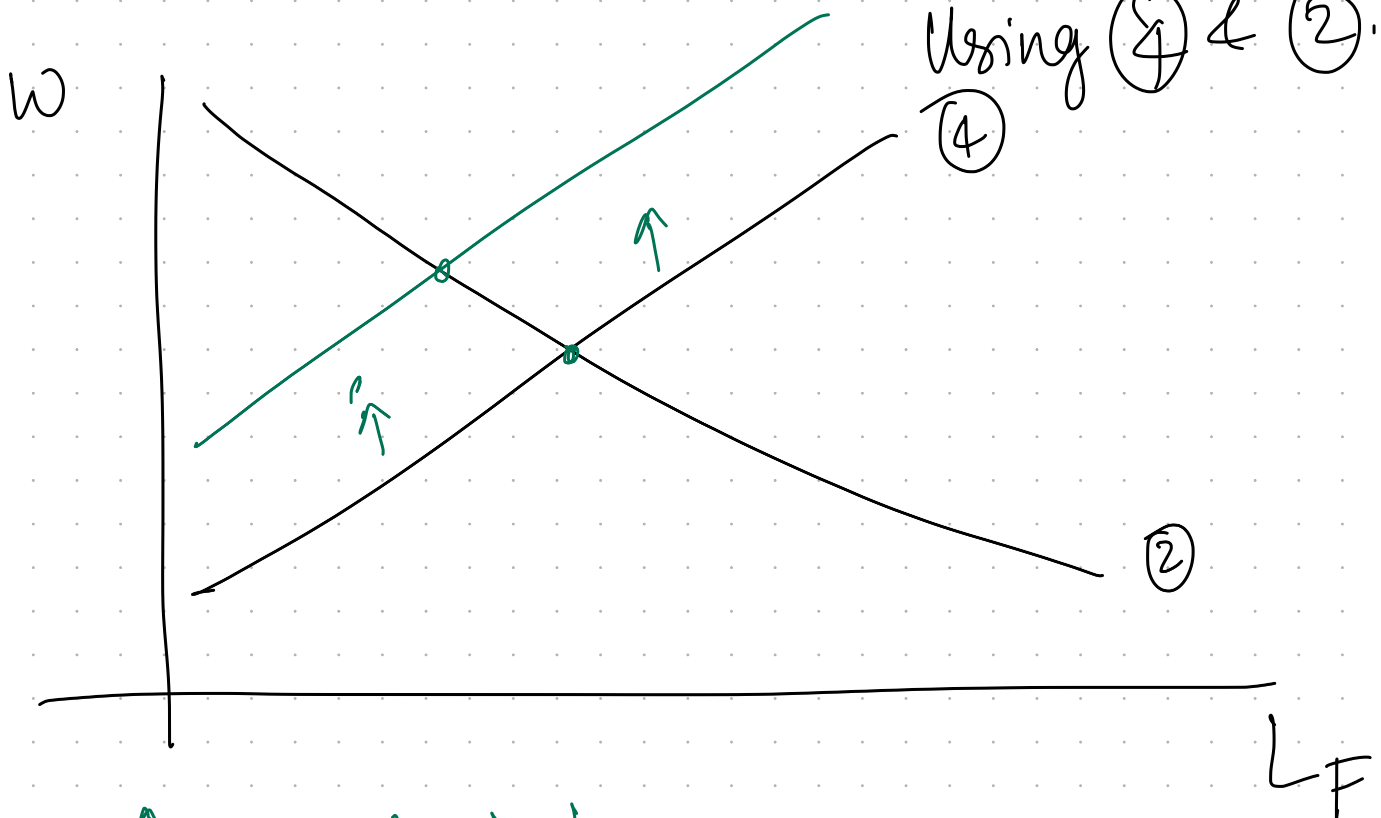
$$\uparrow q(1-b)\left(\frac{K_C}{L_C}\right)^{b} = w \quad \text{—(4)}$$

$L_C \to L - L_F$

MC: $N_F = N$

$K_C = K$

$L_C + L_F = L$



$q \uparrow \Rightarrow w \uparrow, L_F \downarrow$